Axion Quality Problem Alleviated by Non-Minimal Coupling to Gravity

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Based on K. Hamaguchi, Y. Kanazawa, N. Nagata, arXiv:2108.13245.

Outline

- Introduction: Axion Quality Problem
- Wormholes
- Our work
- Summary

Introduction: Axion Quality Problem

Strong CP problem

CP symmetry can be violated in the strong interaction:

$$\mathcal{L}_\theta = \theta_G \frac{\alpha_s}{8\pi} G^A_{\mu\nu} \widetilde{G}^{A\mu\nu} - \sum_q m_q \bar{q} \theta_q i \gamma_5 q \qquad \widetilde{G}^A_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{A\mu\nu}$$

These terms induce neutron electric dipole moment.

$$\Rightarrow \quad d_n = 8.2 \times 10^{-17} \bar{\theta} \; e \cdot \mathrm{cm} \qquad \bar{\theta} = \theta_G + \sum_q \theta_q$$

K. Fuyuto, J. Hisano, N. Nagata, Phys. Rev. **D87**, 054018 (2013).

nEDM collaboration

$$|d_n| < 1.8 \times 10^{-26} \; e \cdot \mathrm{cm} \quad \Rightarrow \quad |\bar{\theta}| \lesssim 10^{-10}$$

Why should it be so small??

Strong CP problem

Peccei-Quinn mechanism

R. D. Peccei and H. R. Quinn (1977):

The strong CP problem can be solved if we introduce a new anomalous global U(1) symmetry.

→ Peccei-Quinn (PQ) symmetry

- This symmetry is spontaneously broken at a scale f_a
- A pseudo Nambu-Goldstone boson appears

→ Axion   S. Weinberg (1978); F. Wilczek (1978).

Peccei-Quinn mechanism

Under the PQ transformation, the axion field shifts as

$$a \to a + f_a \alpha$$

At low energies, the axion field has the following interaction

$$\mathcal{L}_{\rm int} = \frac{g_s^2}{32\pi^2 f_a} a \, G^A_{\mu\nu} \widetilde{G}^{A\mu\nu}$$

(To reproduce anomaly)

After QCD is confined, this interaction induces a potential:

$$V(a) \simeq \frac{1}{2} \mathcal{T} \left(\frac{a}{f_a} + \bar{\theta} \right)^2 \quad \Rightarrow \quad \frac{\langle a \rangle}{f_a} + \bar{\theta} = 0 \qquad m_a^2 = \frac{\mathcal{T}}{f_a^2}$$

$\mathcal{T} \simeq \Lambda_{\rm QCD}^4$: topological susceptibility

Strong CP problem solved.

Achilles' heel of PQ mechanism

The PQ mechanism relies on the assumption that the PQ symmetry is violated solely by QCD non-perturbative effects.

(+ quark masses)

Additional PQ-violating sources may spoil this mechanism.

If their effects are stronger than the QCD effects.

On the other hand, any global symmetries are believed to be violated by gravity.

See, *e.g.*, T. Banks and N. Seiberg, Phys. Rev. D **83**, 084019 (2011).

→ PQ-violating effects by gravity??

Effective theoretical approaches

It was discussed based on effective operator analyses that the gravitational PQ-violating effect can actually be too large.

H. M. Georgi, L. J. Hall, and M. B. Wise (1981); M. Dine and N. Seiberg (1986);
M. Kamionkowski and J. March-Russell (1992); S. M. Barr and D. Seckel (1992);
R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow (1992);
S. Ghigna, M. Lusignoli, and M. Roncadelli (1992).

PQ-violating operator

$$\mathcal{L}_{\rm eff} = \frac{c_n}{M_P^{(n-4)}}\Phi^n + {\rm h.c.}$$

Φ: PQ field

$$\Phi \to \frac{f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

$$\Rightarrow \quad V(a) = -2|c_n| M_P^4 \left(\frac{f_a}{\sqrt{2}M_P}\right)^n \cos\left(\frac{na}{f_a} + \delta_n\right)$$

$$\delta_n \equiv \arg(c_n)$$

This shifts the axion field from the CP-conserving minimum by

$$\frac{|\Delta a|}{f_a} \simeq 2n|c_n \sin\delta_n| \left(\frac{M_P}{\Lambda_{\rm QCD}}\right)^4 \left(\frac{f_a}{\sqrt{2}M_P}\right)^n$$

Axion quality problem

To avoid the strong CP problem, we need $\frac{|\Delta a|}{f_a} \lesssim 10^{-10}$

This means that if $2n\,|\,c_n \sin \delta_n\,| = \mathcal{O}(1)$, we need to forbid the effective operators up to $n \sim 10$ for $f_a \simeq 10^{10}\ \mathrm{GeV}$.

➡ Axion quality problem

Approaches to evade the problem

- Extend the gauge sector.

  Make the PQ symmetry an accidental symmetry.

- Heavy axion models

  Lower the value of f_a

etc.

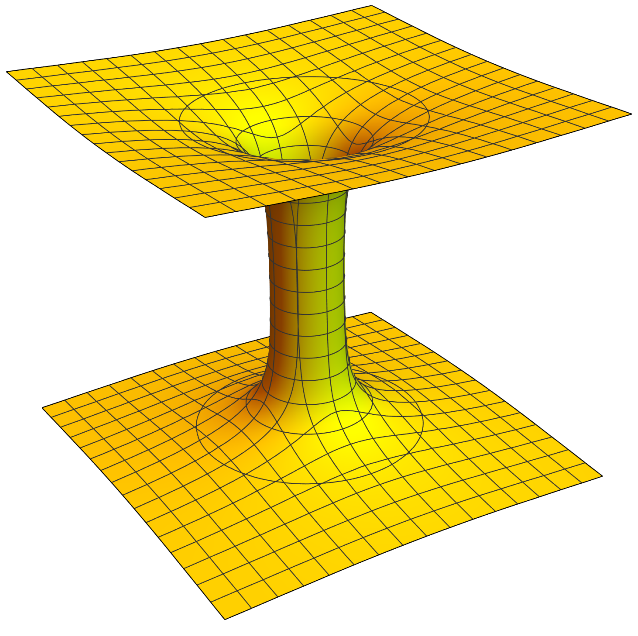
Wormholes

One may wonder what kind of gravitational effects actually cause the violation of the PQ symmetry.

➡ Gravitational instantons or wormholes

Next topic

Wormholes



Giddings-Strominger wormhole

Nuclear Physics B306 (1988) 890–907
North-Holland, Amsterdam

AXION-INDUCED TOPOLOGY CHANGE IN QUANTUM GRAVITY AND STRING THEORY

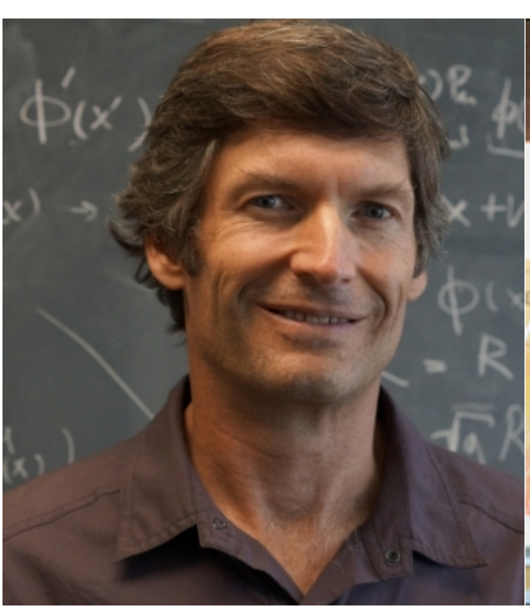
Steven B. GIDDINGS

*Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA, and Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA**

Andrew STROMINGER

Department of Physics, University of California, Santa Barbara, CA 93106, USA

Received 23 October 1987
(Revised 23 November 1987)




They found wormhole solutions of the Euclidean path integral in the theory in which axion minimally couples to gravity.

Giddings-Strominger wormhole

Action

$$S = \int d^4x \sqrt{g} \left[-\frac{M_P^2}{2} R + \frac{f_a^2}{2} g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta \right]$$

Look for a spherically symmetric solution:

$$ds^2 = dr^2 + a(r)^2 d^2\Omega_3$$

$d^2\Omega_3$: 3-dim space element

r: "Euclidean time"

The stationary solutions

- $J^\mu = \sqrt{g}\, g^{\mu\nu} f_a^2 \partial_\nu \theta$ $\quad \partial_\mu J^\mu = 0$ Shift symmetry

$$\Rightarrow \quad n = \int d\Omega_3 \, J^0 = 2\pi^2 a^3(r) f_a^2 \theta'(r) = \text{const.}$$

PQ charge conservation

Giddings-Strominger wormhole

The stationary solutions

- $g_{\mu\nu}$

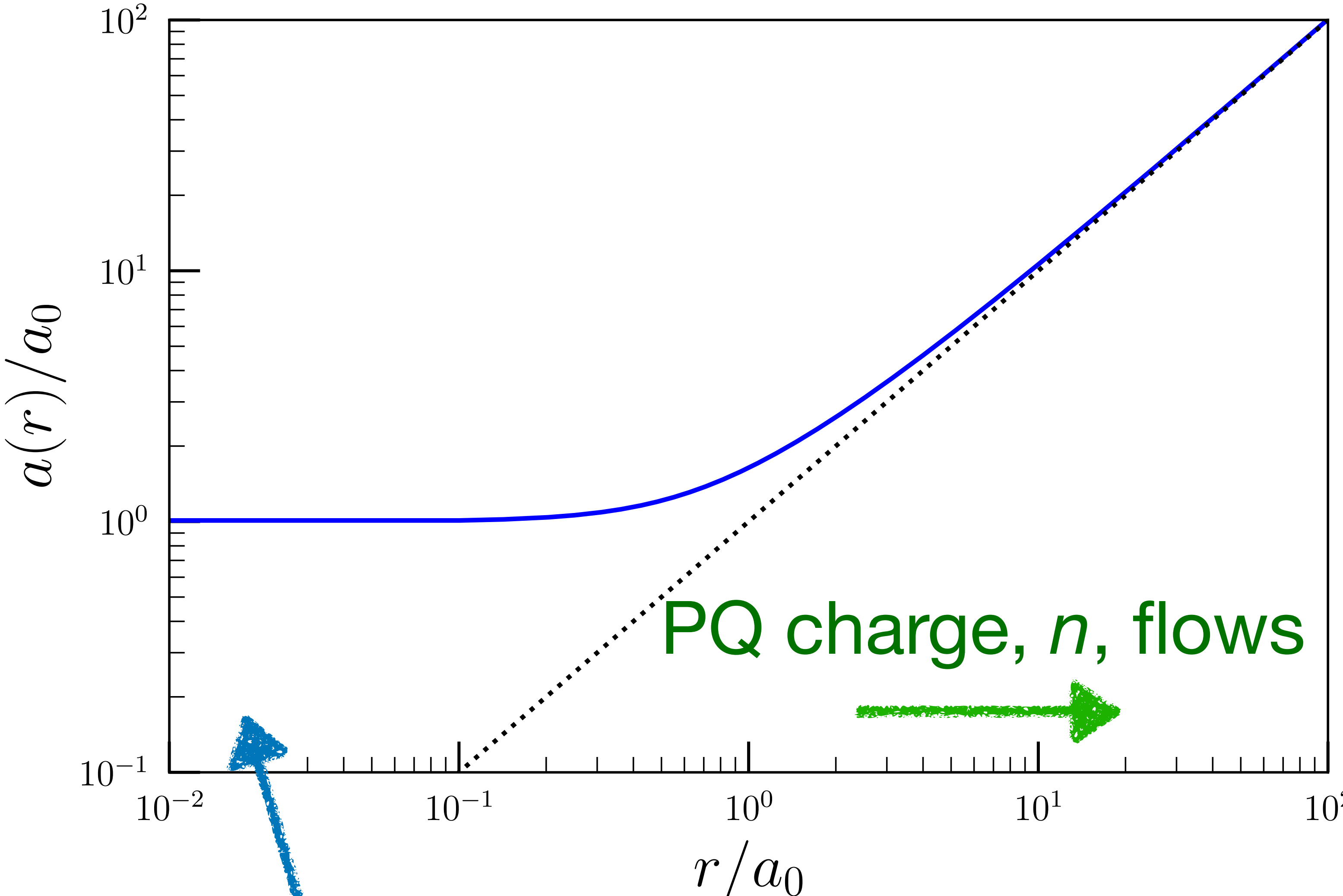
$$3\left(\frac{a'^2}{a^2} - \frac{1}{a^2}\right) = -\frac{f_a^2}{2M_P^2}\theta'^2 \qquad \left(2aa'' + a'^2 - 1\right) = \frac{f_a^2 a^2}{2M_P^2}\theta'^2$$

Does not provide an independent condition.

$$a'^2 = 1 - \frac{a_0^4}{a^4} \qquad a_0 = \left(\frac{n^2}{24\pi^4 f_a^2 M_P^2}\right)^{\frac{1}{4}}$$

The solution can be expressed analytically in terms of elliptic integrals.

Giddings-Strominger wormhole

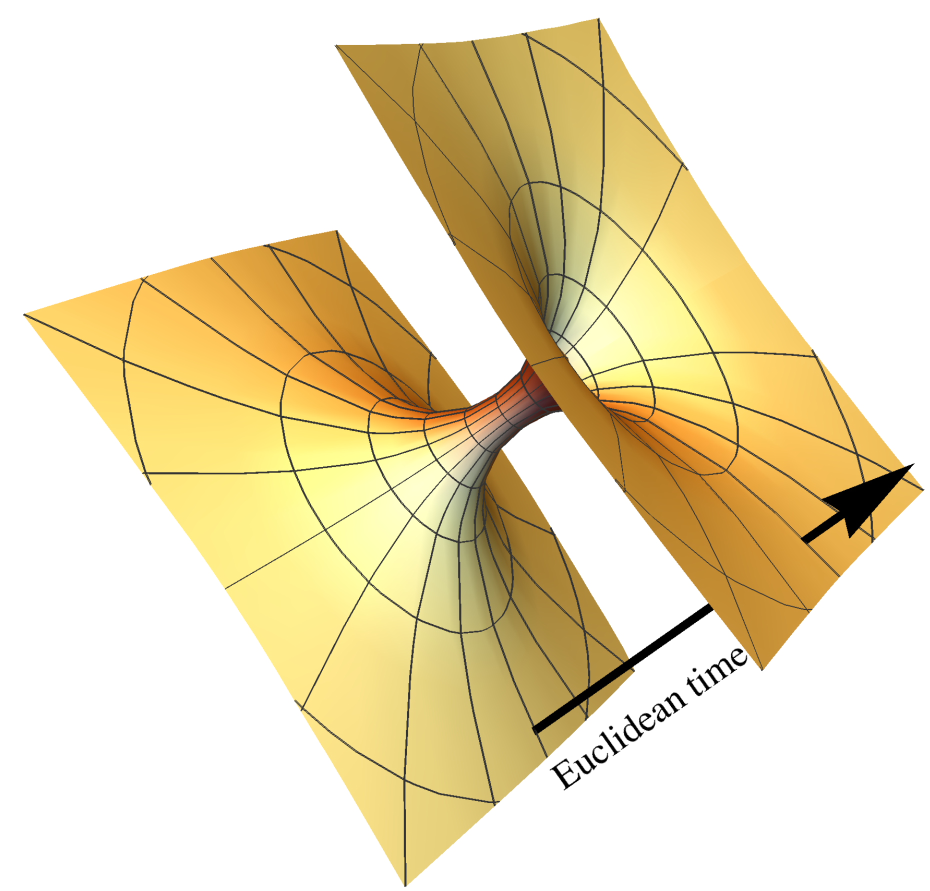


Wormhole throat

$$a(0) = a_0 \quad a'(0) = 0$$

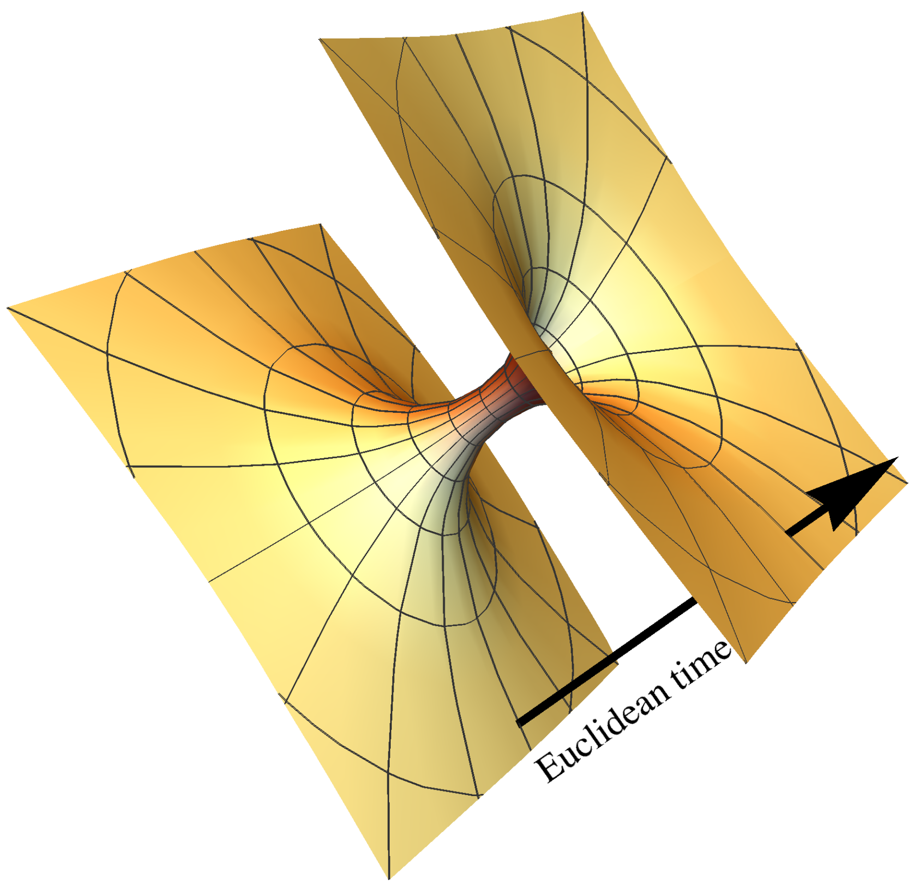
$$a(r) \to r \quad (r \to \infty)$$

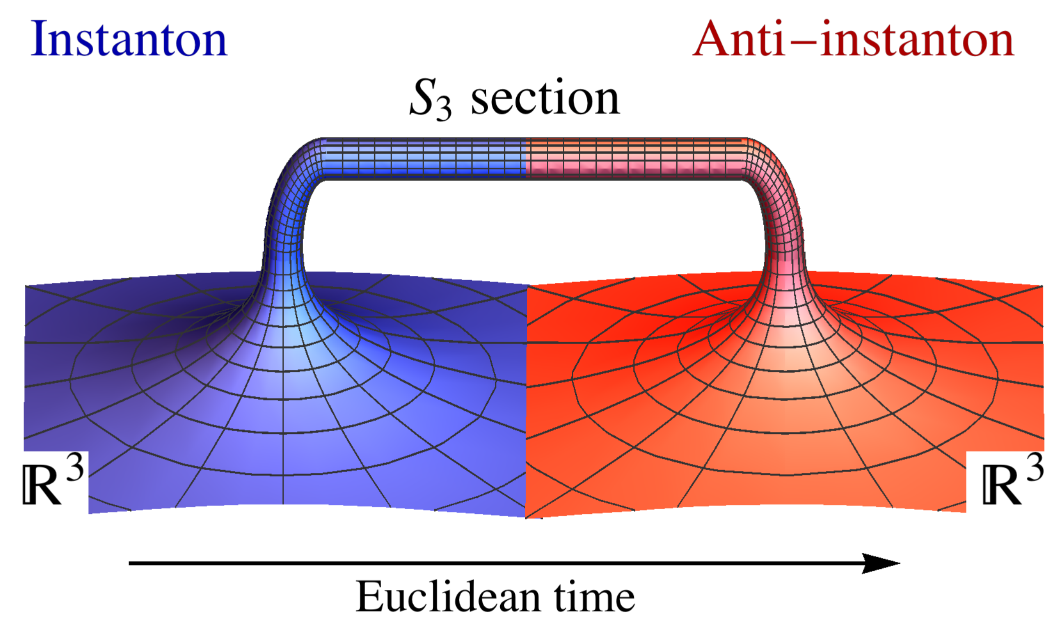
Asymptotically flat



Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

Giddings-Strominger wormhole





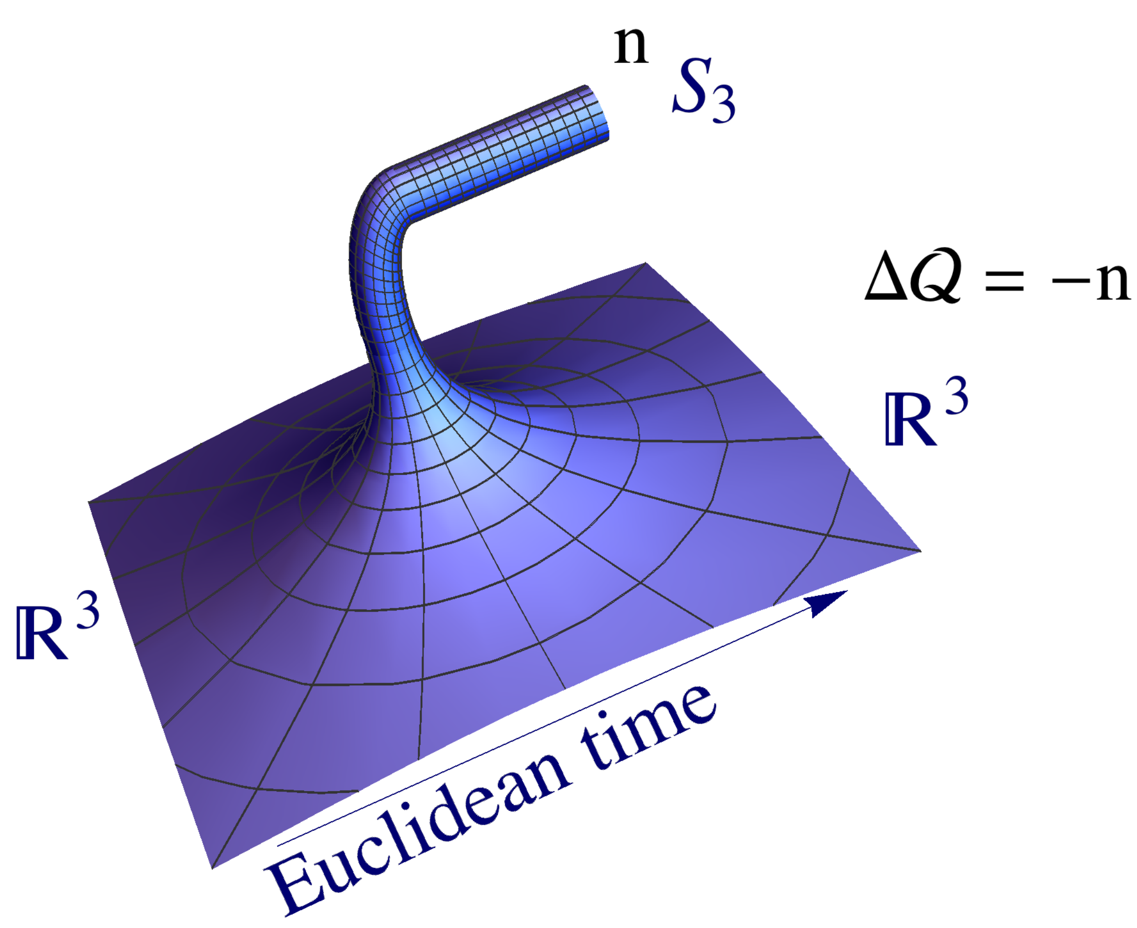
Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

We can also regard two asymptotically flat regions as distinct parts of the same Universe.

A wormhole joins two regions of the same Universe.

$$\mathbb{R}^3 \to \mathbb{R}^3 \oplus S_3 \to \mathbb{R}^3$$

Wormholes as instantons



Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

An observer on $\mathbb{R}^3$ experiences a change in the PQ charge by $\Delta Q = -n$.

➡ For this observer, the PQ charge is not conserved.

Effective potential

These non-perturbative gravitational instantons induce an effective axion potential of the form

S. J. Rey, Phys. Rev. D **39**, 3185 (1989).

$$\Delta V \simeq \frac{1}{a_0^4} e^{-S} \cos\left(\frac{a}{f_a} + \delta\right)$$

This shifts the axion field from the CP-conserving minimum by

$$\frac{|\Delta a|}{f_a} \simeq |\sin\delta| \left(\frac{1}{\Lambda_{\rm QCD}\, a_0}\right)^4 e^{-S} \simeq |\sin\delta| \left(\frac{24\pi^4 f_a^2 M_P^2}{\Lambda_{\rm QCD}^4}\right) e^{-S}$$

For $\delta = \mathcal{O}(1)$, $|\Delta a|/f_a \lesssim 10^{-10}$ is satisfied for

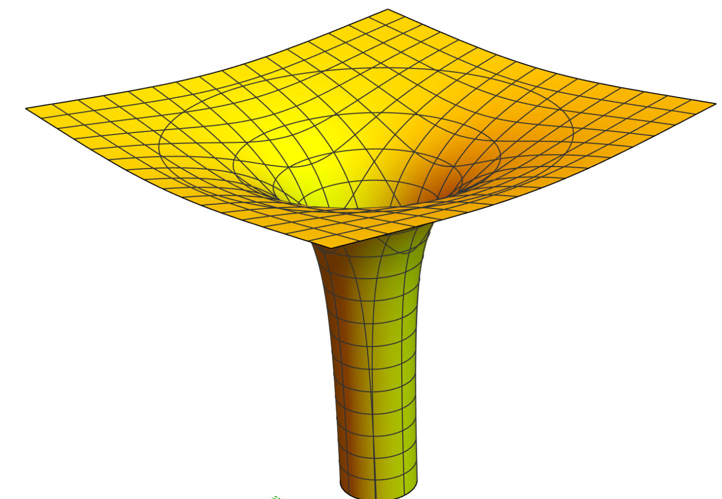
$$S \gtrsim 200$$

Depends only logarithmically on f_a.

Giddings-Strominger wormhole

The value of the action for the wormhole configuration is

$$S = \sqrt{\frac{3\pi^2}{8}}\frac{nM_P}{f_a} - \frac{2}{\pi}\sqrt{\frac{3\pi^2}{8}}\frac{nM_P}{f_a}$$



- This term comes from the boundary contribution.

The Gibbons-Hawking-York (GHY) term.

- Exists only for a semi-wormhole.

For $n = 1$

$$S \simeq 170 \times \left(\frac{f_a}{10^{16}\ \mathrm{GeV}}\right)^{-1}$$

The wormhole contribution is sufficiently suppressed for

$$f_a \lesssim 10^{16}\ \mathrm{GeV}$$

No quality problem!

No quality problem?

Apparently, the wormhole contribution is sufficiently small.

So, we do not need to worry about the quality problem??

→ This turns out to be too optimistic!

In usual axion models, we also have a radial component:

$$\Phi(r) = \frac{f(r)}{\sqrt{2}} e^{i\theta(r)}$$

cf.) $f(r) = f_a$ in Giddings-Strominger.

$f(r)$ has a large field value near the wormhole throat.

L. F. Abbott and M. B. Wise, Nucl. Phys. B **325**, 687 (1989).

→ Modifies the value of the action.

KLLS analysis

PHYSICAL REVIEW D VOLUME 52, NUMBER 2 15 JULY 1995

Gravity and global symmetries

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(Received 17 February 1995)

There exists a widely held notion that gravitational effects can strongly violate global symmetries. If this is correct, it may lead to many important consequences. We argue, in particular, that nonperturbative gravitational effects in the axion theory lead to a strong violation of CP invariance unless they are suppressed by an extremely small factor $g \lesssim 10^{-82}$. One could hope that this problem disappears if one represents the global symmetry of a pseudoscalar axion field as a gauge symmetry of the Ogievetsky-Polubarinov-Kalb-Ramond antisymmetric tensor field. We show, however, that this gauge symmetry does not protect the axion mass from quantum corrections. The amplitude of gravitational effects violating global symmetries could be strongly suppressed by e^{-S}, where S is the action of a wormhole which may absorb the global charge. Unfortunately, in a wide variety of theories based on the Einstein theory of gravity the action appears to be fairly small, $S \sim 10$. However, we find that the existence of wormholes and the value of their action are extremely sensitive to the structure of space on the nearly Planckian scale. We consider several examples (Kaluza-Klein theory, conformal anomaly, R^2 terms) which show that modifications of the Einstein theory on the length scale $l \lesssim 10 M_P^{-1}$ may strongly suppress violation of global symmetries. We find also that in string theory there exists an additional suppression of topology change by the factor $e^{-\frac{8\pi^2}{g^2}}$. This effect is strong enough to save the axion theory for the natural values of the stringy gauge coupling constant.

They found that with the dynamical radial component the action becomes as small as ~ 10.

Quality problem!

KLLS setup

PQ field

$$\Phi(r) = \frac{f(r)}{\sqrt{2}} e^{i\theta(r)}$$

Metric (spherically symmetric)

$$ds^2 = dr^2 + a(r)^2 d^2\Omega_3$$

Action

$$S = \int d^4x \sqrt{g}\left[-\frac{M^2}{2}R + |\partial_\mu \Phi|^2 + V(\Phi)\right] \qquad V(\Phi) = \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$$

The stationary solutions of the path integral can be obtained by minimizing

S. Coleman and K. Lee, Nucl. Phys. B **329**, 387 (1990).

$$S = \int d^4x \sqrt{g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu f)^2 + V(f) + \frac{1}{2gf^2} g_{\mu\nu} J^\mu J^\nu + \frac{1}{\sqrt{g}}\theta\partial_\mu J^\mu\right]$$

with respect to J_μ, f, and $g_{\mu\nu}$.

Stationary solutions

- J_μ

$$J^\mu = \sqrt{g}\, g^{\mu\nu} f^2 \partial_\nu \theta \qquad \partial_\mu J^\mu = 0$$

U(1)$_{PQ}$ conservation

$$\Rightarrow \quad 2\pi^2 a^3(r) f^2(r) \theta'(r) = n$$

- f

$$f'' + 3\frac{a'}{a} f' = \frac{dV}{df} - \frac{n^2}{4\pi^4 f^3 a^6}$$

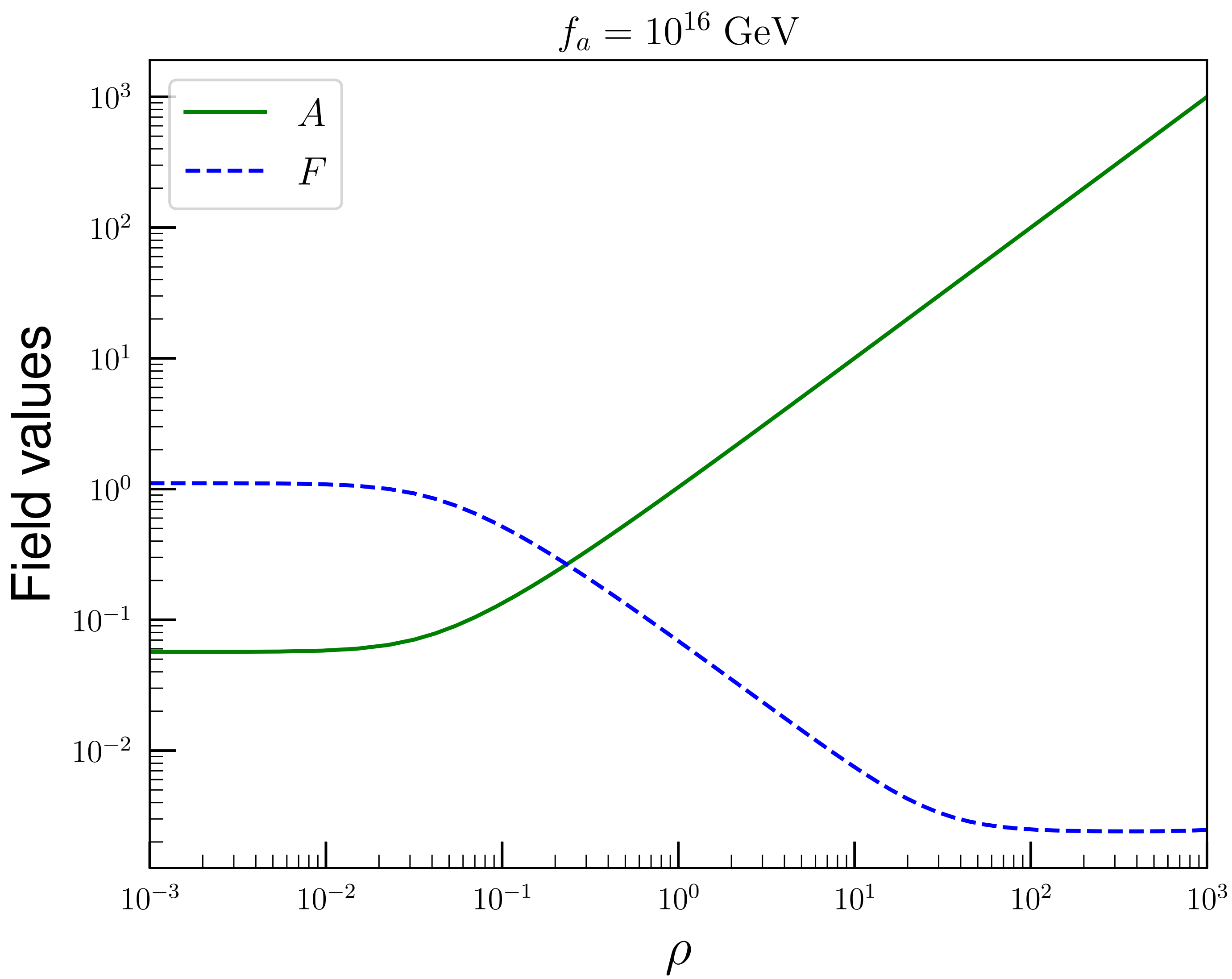
- $g_{\mu\nu}$

$$a'^2 - 1 = -\frac{a^2}{3M_P^2}\left[-\frac{1}{2} f'^2 + V(f) + \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

$$2aa'' + a'^2 - 1 = -\frac{a^2}{M_P^2}\left[\frac{1}{2} f'^2 - \frac{n^2}{8\pi^4 f^2 a^6} + V(f)\right]$$

Again not independent

Results



$$\rho \equiv \sqrt{3\lambda} M_P r$$

$$A \equiv \sqrt{3\lambda}\, M_P a$$

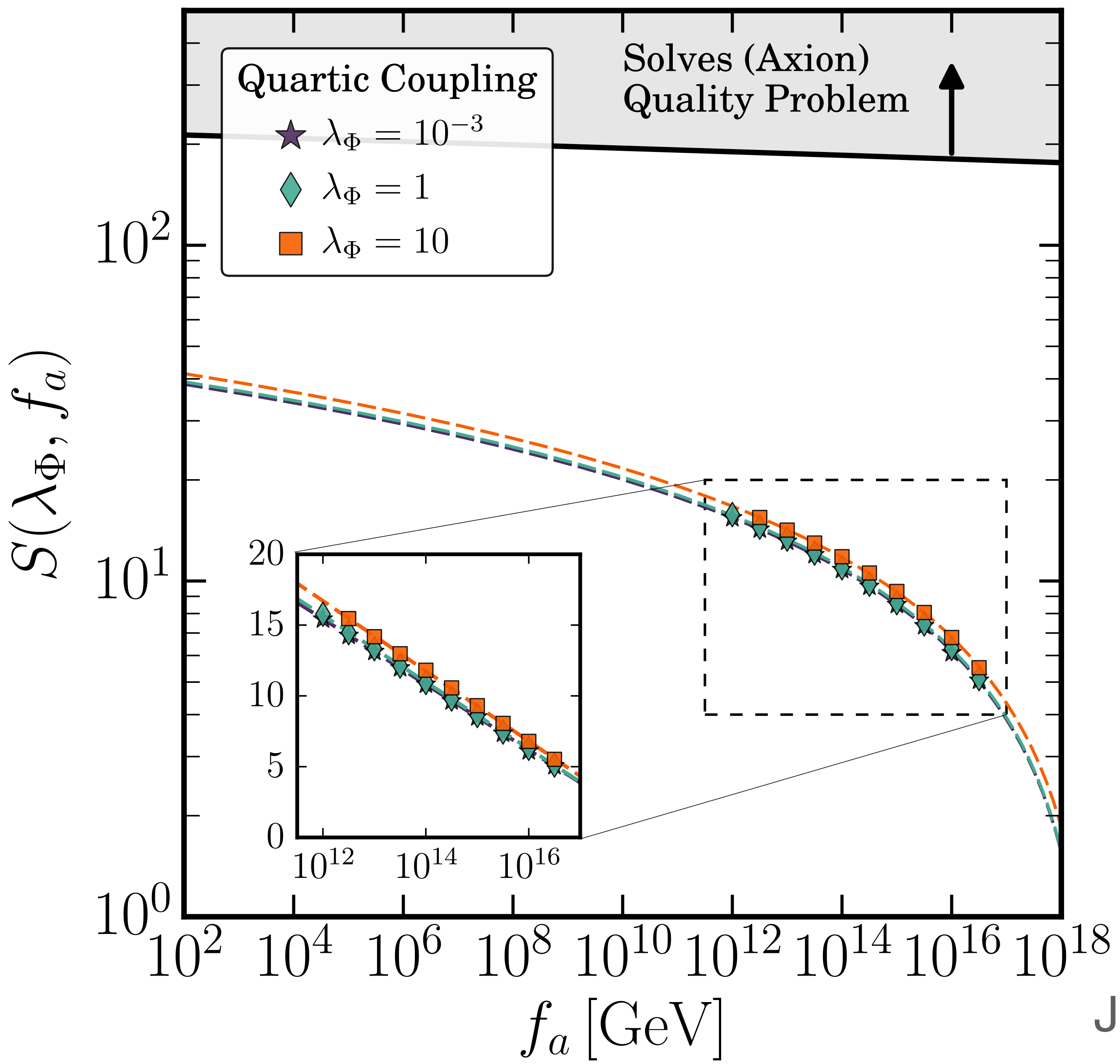
$$F \equiv \frac{f}{\sqrt{3} M_P}$$

Initial conditions

$$f'(0) = 0 \ , \quad f(\infty) = f_a \ , \quad a'(0) = 0$$

$f(r)$ has a large field value near the wormhole throat.

Results



J. Alvey and M. Escudero, JHEP **01**, 032 (2021).

- The value of the axion is much smaller than ~200.
- We have axion quality problem in the minimal setup.

Origin of the difference

The action has the form

$$S = 2\pi^2 \int_0^\infty dr\, a^3(r) \left[\frac{1}{2} f'^2 + \frac{1}{2} f^2 \theta'^2 + \ldots \right]$$

$$= 2\pi^2 \int_0^\infty dr\, a^3(r) \left[\frac{1}{2} f'^2 + \frac{n^2}{8\pi^4 a^6 f^2} + \ldots \right]$$

If f_a is fixed, the second term becomes very large near the throat.

$$\Delta S \simeq 2\pi^2 \cdot a_0^4 \cdot \frac{n^2}{8\pi^4 a_0^6 f_a^2} \sim \frac{n M_P}{f_a}$$

For a dynamical $f(r)$, it can have a value $\sim M_P$ near the throat so that this term remains $\mathcal{O}(1)$.

Summary

- Giddings-Strominger
  - The radial component is fixed.
  - Quality problem can be evaded for $f_a \lesssim 10^{16}$ GeV.
- Kallosh-Linde-Linde-Susskind
  - The radial component is dynamical.
  - Quality problem is still there!

But what happens if we go beyond the minimal setup?

We studied a model in which the axion has a non-minimal coupling to gravity.

K. Hamaguchi, Y. Kanazawa, N. Nagata, arXiv:2108.13245.

Our work

K. Hamaguchi, Y. Kanazawa, N. Nagata, arXiv:2108.13245.

Model

PQ field

$$\Phi(r) = \frac{f(r)}{\sqrt{2}} e^{i\theta(r)}$$

Metric (spherically symmetric)

$$ds^2 = dr^2 + a(r)^2 d^2\Omega_3$$

Action

$$S = \int d^4x \sqrt{g} \left[-\frac{M^2}{2} R - \xi |\Phi|^2 R + |\partial_\mu \Phi|^2 + V(\Phi) \right]$$

Non-minimal coupling

$$V(\Phi) = \lambda \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2$$

In the asymptotically flat regions,

$$|\langle \Phi \rangle| = \frac{f_a}{\sqrt{2}} \qquad M_P^2 = M^2 + \xi f_a^2 \qquad M^2 \geq 0 \;\Rightarrow\; \xi \leq \frac{M_P^2}{f_a^2}$$

Euclidean path integral

The stationary solutions of the path integral can be obtained by minimizing

$$S = \int d^4x \sqrt{g}\left[-\frac{M_P^2}{2}\Omega^2(f)R + \frac{1}{2}(\partial_\mu f)^2 + V(f) + \frac{1}{2gf^2}g_{\mu\nu}J^\mu J^\nu + \frac{1}{\sqrt{g}}\theta\partial_\mu J^\mu\right]$$

with respect to J_μ, f, and $g_{\mu\nu}$.

$$\Omega^2(f) \equiv 1 + \frac{\xi}{M_P^2}\left(f^2 - f_a^2\right)$$

- J_μ

$$J^\mu = \sqrt{g}\, g^{\mu\nu} f^2 \partial_\nu \theta \qquad \partial_\mu J^\mu = 0$$

$U(1)_{PQ}$ conservation

$$\Rightarrow \quad 2\pi^2 a^3(r) f^2(r)\theta'(r) = n$$

Stationary solutions

- f

$$f'' + 3\frac{a'}{a}f' = \frac{dV}{df} - \frac{n^2}{4\pi^4 f^3 a^6} + 6\left[\frac{a''}{a} + \frac{a'^2}{a^2} - \frac{1}{a^2}\right]\xi f$$

- $g_{\mu\nu}$

$$\Omega^2(f)\left(a'^2 - 1\right) + \frac{2\xi}{M_P^2}aa'ff' = -\frac{a^2}{3M_P^2}\left[-\frac{1}{2}f'^2 + V(f) + \frac{n^2}{8\pi^4 f^2 a^6}\right]$$

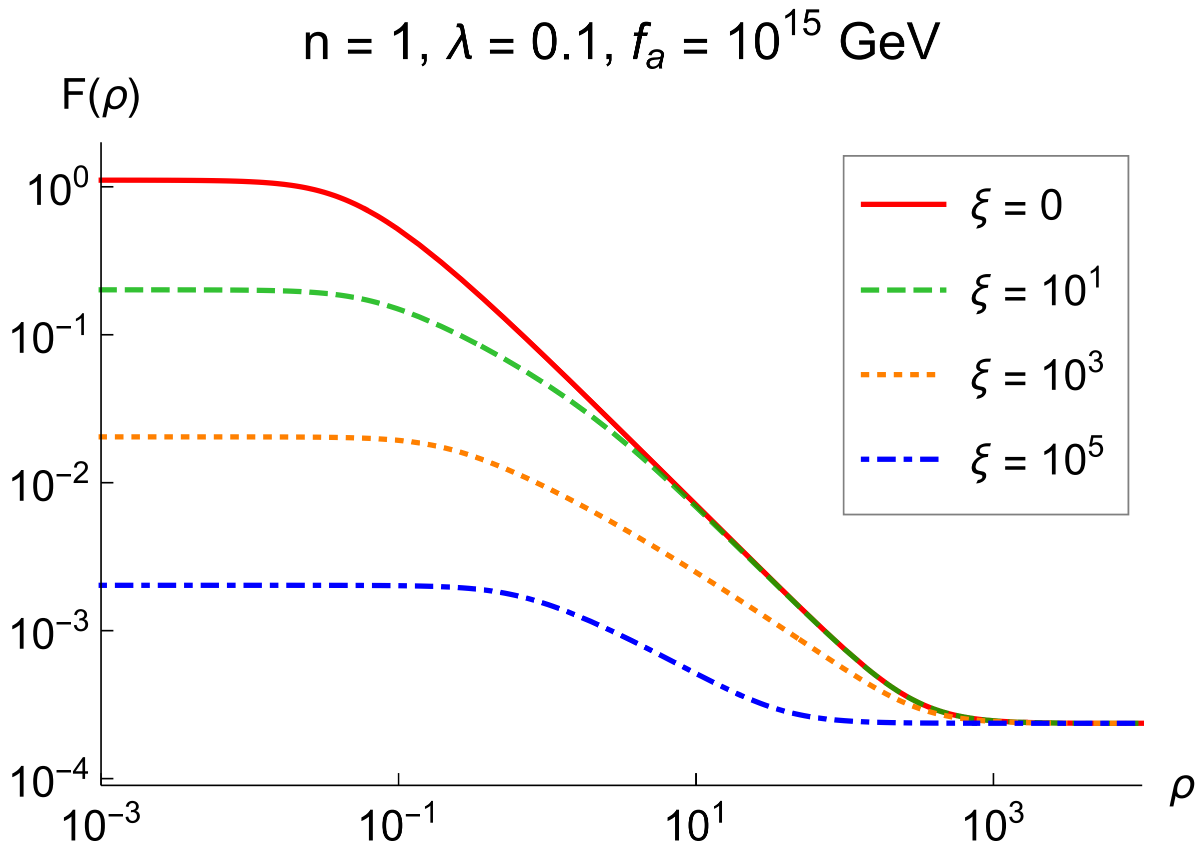
$$\Omega^2(f)\left(2aa'' + a'^2 - 1\right) + \frac{2\xi a^2}{M_P^2}\left[ff'' + f'^2 + 2\frac{a'}{a}ff'\right] = -\frac{a^2}{M_P^2}\left[\frac{1}{2}f'^2 - \frac{n^2}{8\pi^4 f^2 a^6} + V(f)\right]$$

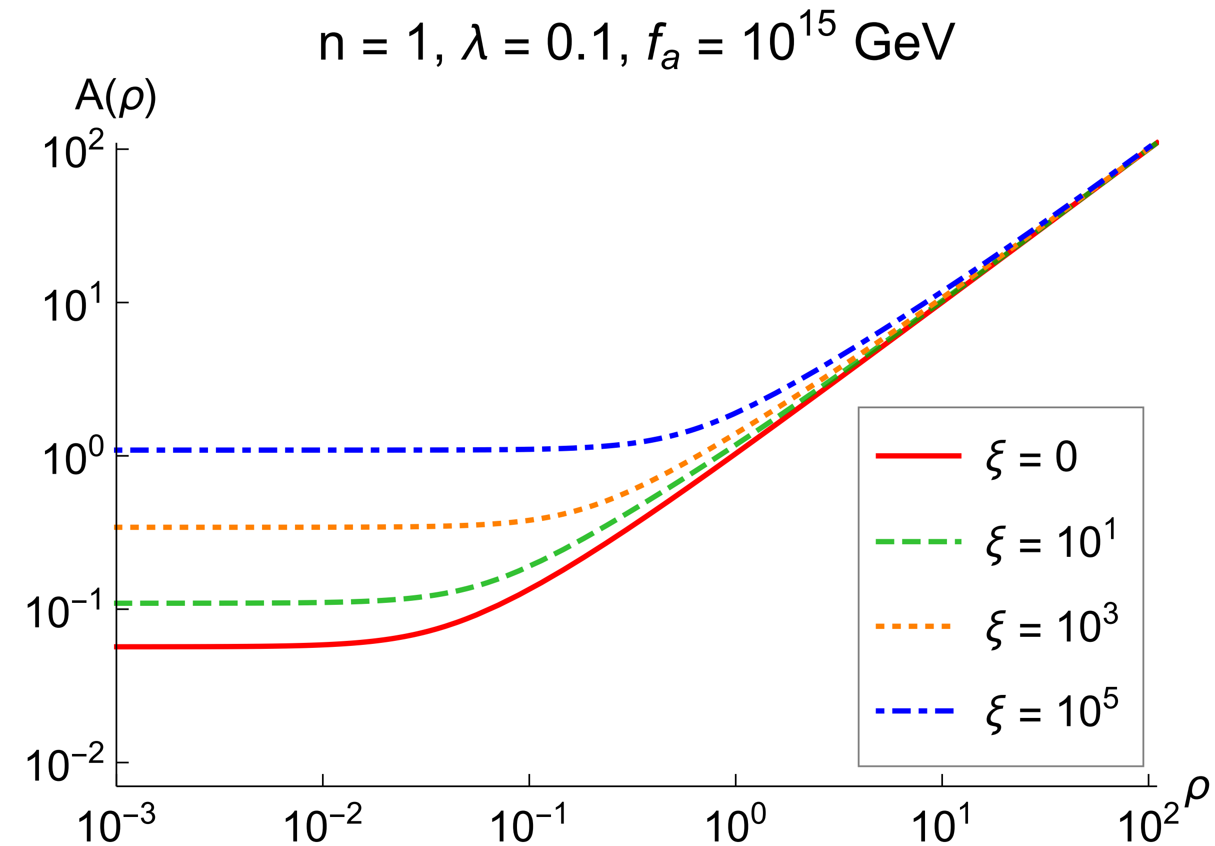
The last equation follows from the first two: two independent Eqs.

Initial conditions

$$f'(0) = 0\ , \quad f(\infty) = f_a\ , \quad a'(0) = 0$$

Results





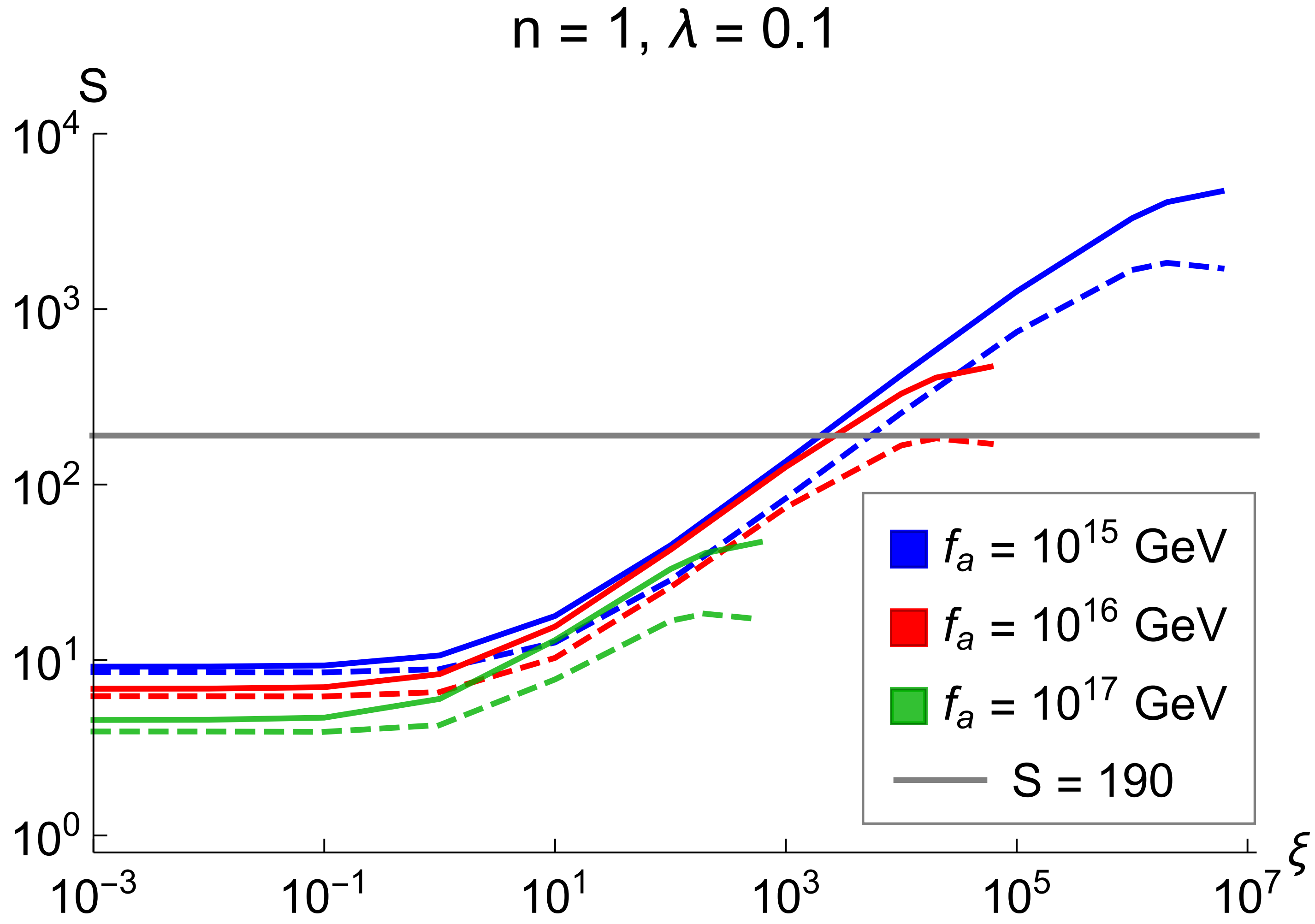
$$\rho \equiv \sqrt{3\lambda}\, M_P r \ , \quad A \equiv \sqrt{3\lambda}\, M_P a \ , \quad F \equiv \frac{f}{\sqrt{3} M_P}$$

For a larger value of ξ

- F(ρ) decreases
- A(ρ) increases

around $\rho \simeq 0$.

Action



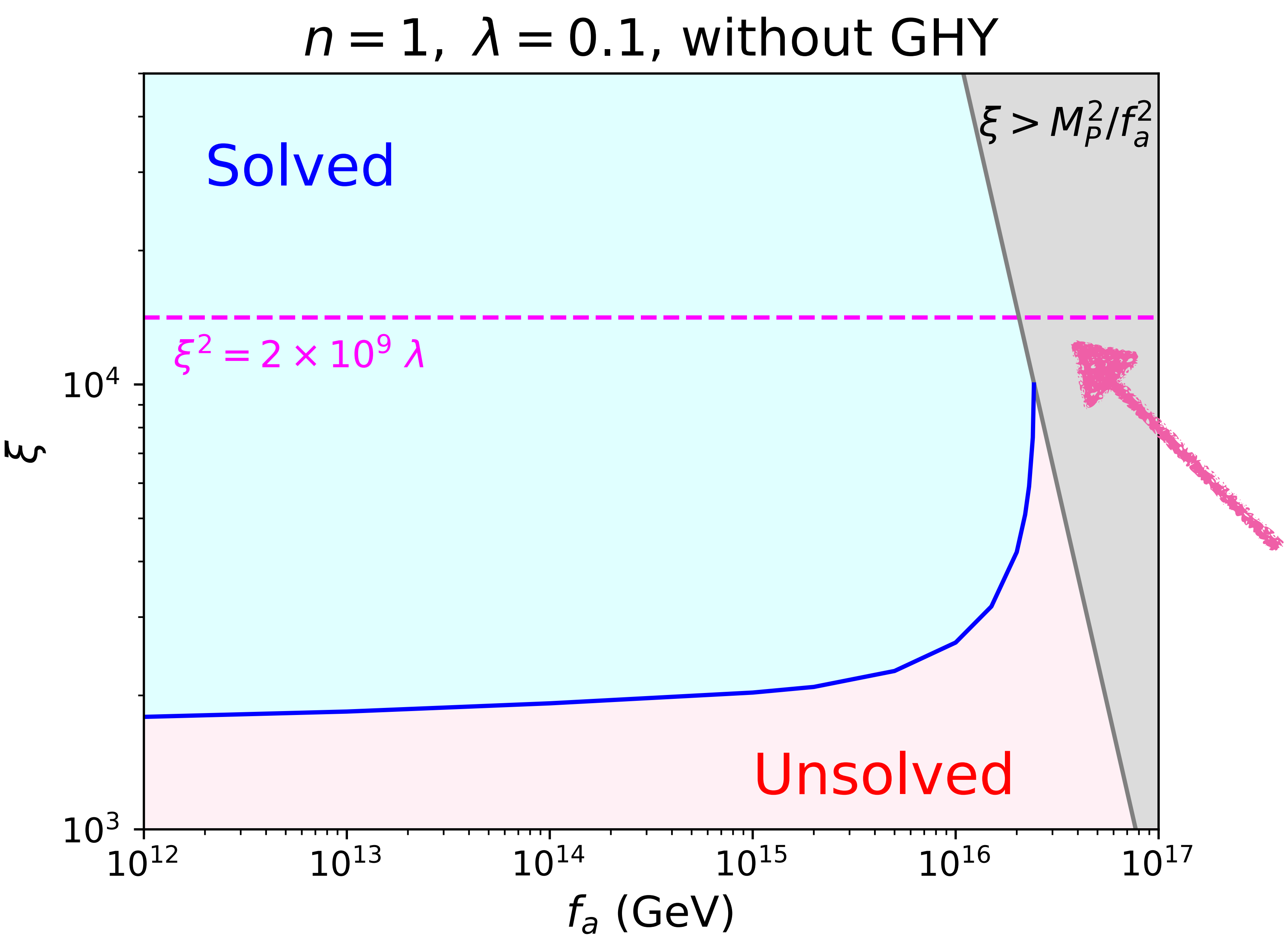
$\Delta a/f_a \lesssim 10^{-10}$

Only $\xi \leq M_P^2/f_a^2$ shown.

- The value of the action significantly increases as ξ increases.
- Quality problem can be evaded for $\xi \gtrsim 2 \times 10^3$.

K. Hamaguchi, Y. Kanazawa, N. Nagata, arXiv:2108.13245.

Results



Successful inflation

$$\xi |\Phi|^2 R$$

- Quality problem can be evaded for $f_a \lesssim 2.5 \times 10^{16}$ GeV.
- This solution may be used in inflation models.

K. Hamaguchi, Y. Kanazawa, N. Nagata, arXiv:2108.13245.

Discussion

At the large end of $\xi = M_P^2/f_a^2$, we find that $f_a =$ const. is the solution.

➡ Giddings-Strominger wormhole — No quality problem

The limit of $\xi = 0$:

➡ KLLS wormhole — Quality problem exists.

The non-minimal coupling ξ smoothly connects these two cases.

Summary

Summary

- Wormholes cause the axion quality problem.
- We found that the non-minimal gravitational coupling can sufficiently suppress the wormhole contribution.
- The same coupling may be useful for inflation.
- Our result points to a new way of model building to avoid the quality problem.

  Such as the modification of the gravitational sector.

Backup

Peccei-Quinn mechanism

The relevant Lagrangian terms are

$$\mathcal{L}_{\rm int} = \frac{g_s^2}{32\pi^2}\left(\frac{a}{f_a} + \theta_G\right) G^A_{\mu\nu}\widetilde{G}^{A\mu\nu} - \sum_q \left[m_q e^{i\theta_q}\bar{q}_R q_L + \text{h.c.}\right]$$

Chiral rotations $q \to e^{-i\gamma_5 \alpha_q/2} q$ shift the first term through anomaly:

$$\Delta\mathcal{L} = -\frac{g_s^2}{32\pi^2}\left(\sum_q \alpha_q\right) G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$

Thus, by taking

$$\sum_q \alpha_q = \frac{a}{f_a} + \theta_G$$

$$\sum_q \left[m_q e^{i(\theta_q + \alpha_q)}\bar{q}_R q_L + \text{h.c.}\right]$$

We can move the coefficient of the first term into the second term.

Chiral Lagrangian

At low energies, the interactions of mesons can be described by the following effective Lagrangian (at the LO):

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)$$

(f_π: pion decay constant)

where

$$U \equiv e^{\frac{2i\pi^a(x)T^a}{f_\pi}}$$

(T^a: $\mathrm{SU}(N_f)$ generators)

Under $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ pion fields transform as

$$U \to g_R U g_L^{-1}$$

$$g_L \in \mathrm{SU}(N_f)_L \quad g_R \in \mathrm{SU}(N_f)_R$$

The above Lagrangian is invariant under this transformation.

Mass term

The effect of the mass term can be included in the low-energy effective Lagrangian as follows:

$$\mathcal{L}_{\rm mass} = -\sum_q \left[m_q e^{i(\theta_q+\alpha_q)} \bar{q}_R q_L + \text{h.c.} \right] \qquad (N_f = 2)$$

$$= -\underbrace{(\bar{u}_R,\, \bar{d}_R)}_{\bar{q}_R} \underbrace{\begin{pmatrix} m_u e^{i(\theta_u+\alpha_u)} & 0 \\ 0 & m_d e^{i(\theta_d+\alpha_d)} \end{pmatrix}}_{\mathcal{M}} \underbrace{\begin{pmatrix} u_L \\ d_L \end{pmatrix}}_{q_L} + \text{h.c.}$$

Transformation laws

$$q_L \to g_L q_L \qquad q_R \to g_R q_R$$

The above term would be invariant if $\mathcal{M}$ transformed as

$$\mathcal{M} \to g_R \mathcal{M} g_L^{-1}$$

Mass term

We can thus include the effect of $\mathscr{M}$ in an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ invariant manner as

$$\mathcal{L}_{\mathrm{mass}} = \frac{f_\pi^2 B_0}{2} \mathrm{Tr}\left(U\mathcal{M}^\dagger + \mathcal{M}U^\dagger\right)$$

(B_0: a constant)

Let us express this in terms of the pion fields.

$$U \equiv e^{\frac{2i\pi^a(x)T^a}{f_\pi}} \qquad 2\pi^a T^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

We focus on the neutral sector to obtain the axion potential.

Mass term

$$\mathcal{L}_{\rm mass} = f_\pi^2 B_0 {\rm Re}\left[e^{i\frac{\pi^0}{f_\pi}}\left\{m_u e^{-i(\theta_u+\alpha_u)} + m_d e^{i(\theta_d+\alpha_d)}\right\}\right]$$

$$= f_\pi^2 B_0 (m_u + m_d)\left[1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left\{\frac{1}{2}\left(\frac{a}{f_a} + \bar\theta\right)\right\}\right]^{\frac{1}{2}} \cos\left(\frac{\pi^0}{f_\pi} - \phi\right)$$

where

$$\bar\theta = \theta_G + \sum_q \theta_q \qquad \tan\phi \equiv \frac{m_u \sin(\theta_u+\alpha_u) - m_d \sin(\theta_d+\alpha_d)}{m_u\cos(\theta_u+\alpha_u) + m_d\cos(\theta_d+\alpha_d)}$$

We find that the pion mass is given by

$$m_\pi^2 = B_0(m_u + m_d)$$

The pion field acquires a VEV to minimize the potential: $\langle \pi^0 \rangle = f_\pi \phi$

Axion potential

By integrating out the pion field, we obtain the axion potential

$$V(a) = -f_\pi^2 m_\pi^2 \left[1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left\{\frac{1}{2}\left(\frac{a}{f_a}+\bar{\theta}\right)\right\}\right]^{\frac{1}{2}}$$

At the minimum,

$$\frac{\langle a\rangle}{f_a} + \bar{\theta} = 0$$

Strong CP problem is solved.

Axion mass is given by

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{f_\pi^2 m_\pi^2}{f_a^2}$$

$$m_a \simeq 6\ \mathrm{meV} \times \left(\frac{10^9\ \mathrm{GeV}}{f_a}\right)$$

Folk theorem

Suppose that particles carrying global charges are swallowed by a black hole.

The BH eventually evaporates by emitting Hawking radiation.

➡ The global charges are destroyed.

Violation by gravity

Cf.) Gauge charges

Gauss law ensures that the electric flux is preserved.

Charged BHs cannot evaporate entirely.

Two-form gauge theory

In the original paper by S. Giddings and A. Strominger, a two-form gauge theory was considered.

$b_{\mu\nu}$: Kalb-Ramond field

Field strength

$$H = db \qquad H_{\mu\nu\rho} = \nabla_\mu b_{\nu\rho} + \nabla_\nu b_{\rho\mu} + \nabla_\rho b_{\mu\nu}$$

Gauge transformations

$$\delta b = d\Lambda \qquad \delta b_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

H is dual of the axion field θ:

$$H_{\mu\nu\rho} = f_a^2 \epsilon_{\lambda\mu\nu\rho} \partial^\lambda \theta$$

Two-form gauge theory

Action

$$S = \int d^4x \sqrt{g}\left(-\frac{M_P^2}{2}R + \frac{1}{12f_a^2}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

Look for a spherically symmetric solution:

$$ds^2 = dr^2 + a(r)^2 d^2\Omega_3$$

$$H_{\mu\nu\rho} = \mathcal{H}(r)\epsilon_{ijk}$$

$d^2\Omega_3$: 3-dim space element

r: "Euclidean time"

The stationary solutions

- $g_{\mu\nu}$

$$3\left[\frac{a'^2}{a^2} - \frac{1}{a^2}\right] = -\frac{1}{2f_a^2 M_P^2}\mathcal{H}^2 \qquad 2aa'' + a'^2 - 1 = \frac{1}{2f_a^2 M_P^2}\mathcal{H}^2 a^2$$

Two-form gauge theory

The stationary solutions

- $H_{\mu\nu\rho}$

$$\partial_\mu(\sqrt{g}H^{\mu\nu\rho}) = 0$$

Always satisfied under the ansatz.

- Bianchi identity

$$dH = 0 \qquad \epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{\nu\rho\sigma} = 0$$

$$\Rightarrow \quad \frac{d}{dr}\left(\sqrt{|\widetilde{g}|}a^3(r)\mathcal{H}(r)\varepsilon_{ijk}\right) = 0 \quad \Rightarrow \quad \mathcal{H}(r) = \frac{\mathcal{H}_0}{a^3(r)}$$

We normalize $\mathcal{H}_0$ at $r \to \infty$ as

$$\int d\Omega_3\, r^3\mathcal{H}(r) = \int d\Omega_3\, \mathcal{H}_0 = 2\pi^2\mathcal{H}_0 = n$$

Giddings-Strominger wormhole

The Giddings-Strominger wormhole solution can be expressed analytically in terms of elliptic integrals:

$$r = a\sqrt{1-\frac{a_0^4}{a^4}} + \frac{a_0}{\sqrt{2}} F\left(\cos^{-1}\left\{\frac{a_0}{a}\right\}\middle|\frac{1}{2}\right) - \sqrt{2}a_0 E\left(\cos^{-1}\left\{\frac{a_0}{a}\right\}\middle|\frac{1}{2}\right)$$

S. B. Giddings and A. Strominger, Nucl. Phys. B **306**, 890 (1988).

Elliptic integrals

$$F(\phi|m) = \int_0^\phi \left(1 - m\sin^2\theta\right)^{-1/2} d\theta \ ,$$

$$E(\phi|m) = \int_0^\phi \left(1 - m\sin^2\theta\right)^{1/2} d\theta \ .$$

Charge quantization

$$n = \int d\Omega_3\, J^0 = 2\pi^2 a^3(r) f_a^2 \theta'(r)$$ is an integer.

To see this, consider the axion part of the action in the Lorentzian spacetime:

e^{iS_θ}

$$S_\theta = \int d^4x\,\sqrt{g}\,\frac{f_a^2}{2}\dot{\theta}^2 = \int dt \int d\Omega_3 \sqrt{g_3}\,\frac{f_a^2}{2}\dot{\theta}^2$$

For a shift $\theta \to \theta + \delta\theta$,

Note that $\frac{d}{dt}(\sqrt{g_3}\,\dot{\theta}) = 0$

$$\delta S_\theta = f_a^2 \int dt \int d\Omega_3 \sqrt{g_3}\,\dot{\theta}\delta\dot{\theta}$$

$$\Rightarrow \quad \delta S_\theta = f_a^2 \left[\int d\Omega_3 \sqrt{g_3}\,\dot{\theta}\delta\theta\right]_{\text{boundary}} = n\,[\delta\theta]_{\text{boundary}}$$

But since $\theta \to \theta + 2\pi k$ is a gauge redundancy, $n \in \mathbb{Z}$.

Gibbons-Hawking-York term

In the presence of a boundary, the action must be supplemented by a boundary term so that the variational principle is well-defined.

$$S_{\mathrm{GHY}} = -M_P^2 \int_{\partial V} d^3x \sqrt{|\tilde{g}|}\, K$$

with

$$K \equiv g^{\mu\nu} K_{\mu\nu} = P^{\alpha\beta} \nabla_\alpha n_\beta$$

- $K_{\mu\nu}$: the extrinsic curvature
- $P_{\mu\nu} \equiv g_{\mu\nu} - n_\mu n_\nu$: the projection tensor
- n^μ: unit normal vector of the hypersurface

This is called the Gibbons-Hawking-York (GHY) term.

Gibbons-Hawking-York term

In the present case, we have

$$K = P^{\alpha\beta}\nabla_\alpha n_\beta = g^{ij}\nabla_i n_j = g^{ij}(-\Gamma^0_{ij}) = aa'g^{ij}\frac{g_{ij}}{a^2} = 3\frac{a'}{a}$$

and thus

$$S_{\mathrm{GHY}} = -3M_P^2\int_{\partial V} d\Omega_3 a^2 a'$$

Notice that the integrand is divergent. The standard way to overcome this issue is

$$K \to K - K_0$$

K_0: extrinsic curvature of the same boundary embedded in flat spacetime.

$$K_0 = \frac{3}{a}$$

Caveat

There are subtleties in finding stationary solutions for axionic wormholses, related with the PQ charge conservation.

K. Lee, Phys. Rev. Lett. **61**, 263 (1988).

The origin of the issue

S. Coleman and K. Lee, Nucl. Phys. B **329**, 387 (1990).

Consider the calculation of the lowest energy of some fixed value of a conserved charge Q.

$$\langle f|e^{-HT/\hbar}|i\rangle = \sum_n e^{-E_n T/\hbar}\langle f|n\rangle\langle n|i\rangle$$

We cannot directly use the saddle-point approximation since we are concerned with excited states.

Instead, we should compute

$$\langle f|e^{-HT/\hbar}\delta(Q-q)|i\rangle = \sum_n e^{-E_n T/\hbar}\delta(Q_n - q)\langle f|n\rangle\langle n|i\rangle$$

Caveat

Two prescriptions to overcome this issue in the present case were discussed in S. Coleman and K. Lee, Nucl. Phys. B **329**, 387 (1990).

- Minimize the following action:

$$S = \int d^4x \sqrt{g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu f)^2 + V(f) + \frac{1}{2gf^2}g_{\mu\nu}J^\mu J^\nu + \frac{1}{\sqrt{g}}\theta\partial_\mu J^\mu\right]$$

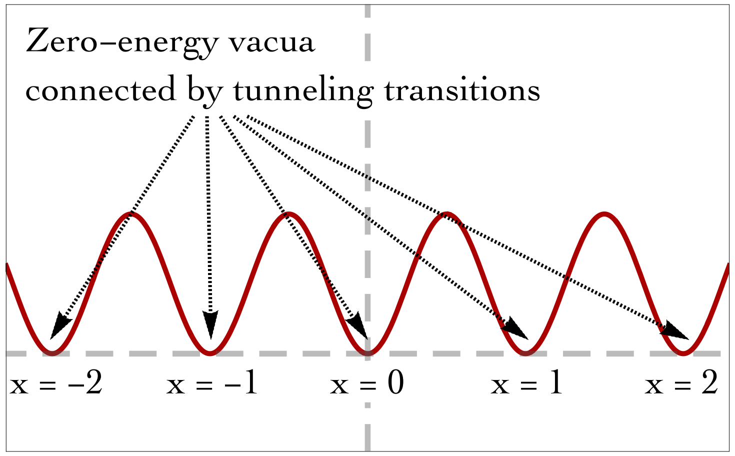
  with respect to J_μ, f, and $g_{\mu\nu}$.

  - Can be derived from the phase-space path integral.
  - The conservation law is taken into account with a Lagrange multiplier.

- Look for stationary points for imaginary θ

$$\phi = i\theta$$

1-D periodic potential



Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

With the dilute-gas approximation, we have

$$\langle k|e^{-HT}|j\rangle = \sqrt{\frac{\omega}{\pi}}e^{-\omega T/2}\sum_{n=0}^{\infty}\sum_{\bar{n}=0}^{\infty}\frac{1}{n!\bar{n}!}\left(Ke^{-S_0}T\right)^{n+\bar{n}}\delta_{n-\bar{n},k-j}$$

$$\omega^2 = V''(0) \quad K \equiv \sqrt{\frac{S_0}{2\pi}}\left|\frac{\det(-\partial_t^2+\omega^2)}{\det'(-\partial_t^2+V''(\bar{x})}\right|^{1/2}$$

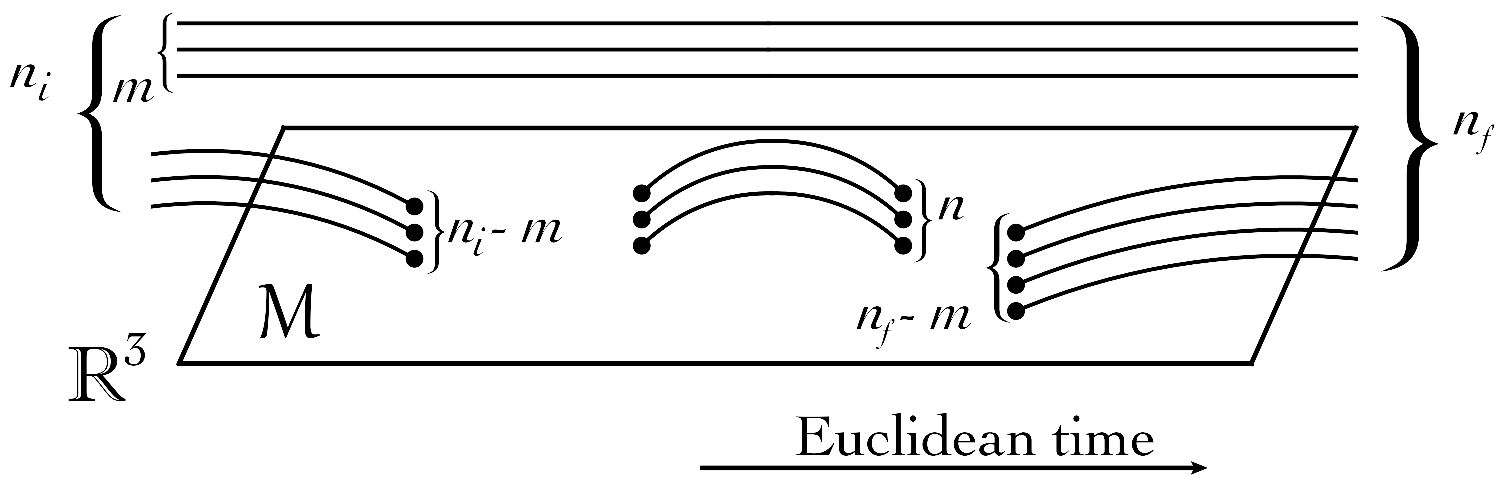
This can be computed as

$$\langle k|e^{-HT}|j\rangle = \sqrt{\frac{\omega}{\pi}}e^{-\omega T/2}\int_0^{2\pi}\frac{d\theta}{2\pi}e^{i\theta(j-k)}\exp(2KTe^{-S_0}\cos\theta)$$

Energy eigenstates

$$|\theta\rangle = \left(\frac{\omega}{\pi}\right)^{1/4}\frac{1}{\sqrt{2\pi}}\sum_n e^{-i\theta n}|n\rangle \qquad E(\theta) = \frac{\omega}{2} - 2Ke^{-S_0}\cos\theta$$

Multiple wormholes



Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

The amplitude for the transition between n_i and n_f instanton states is given by

S. B. Giddings and A. Strominger, Nucl. Phys. B **307**, 854 (1988).

$$\langle n_f | e^{-HT} | n_i \rangle = \sum_{n=0}^{\infty} \sum_{m=0}^{\min(n_i,n_f)} \frac{\sqrt{n_i! n_f!}}{m!} \frac{(KVTe^{-S})^{2n+n_i+n_f-2m}}{2^n n!(n_i-m)!(n_f-m)!}$$

The same result can be obtained using the Hamiltonian

$$H = Ke^{-S}V(a+a^\dagger)$$

a: instanton annihilation operator

Effective wormhole action

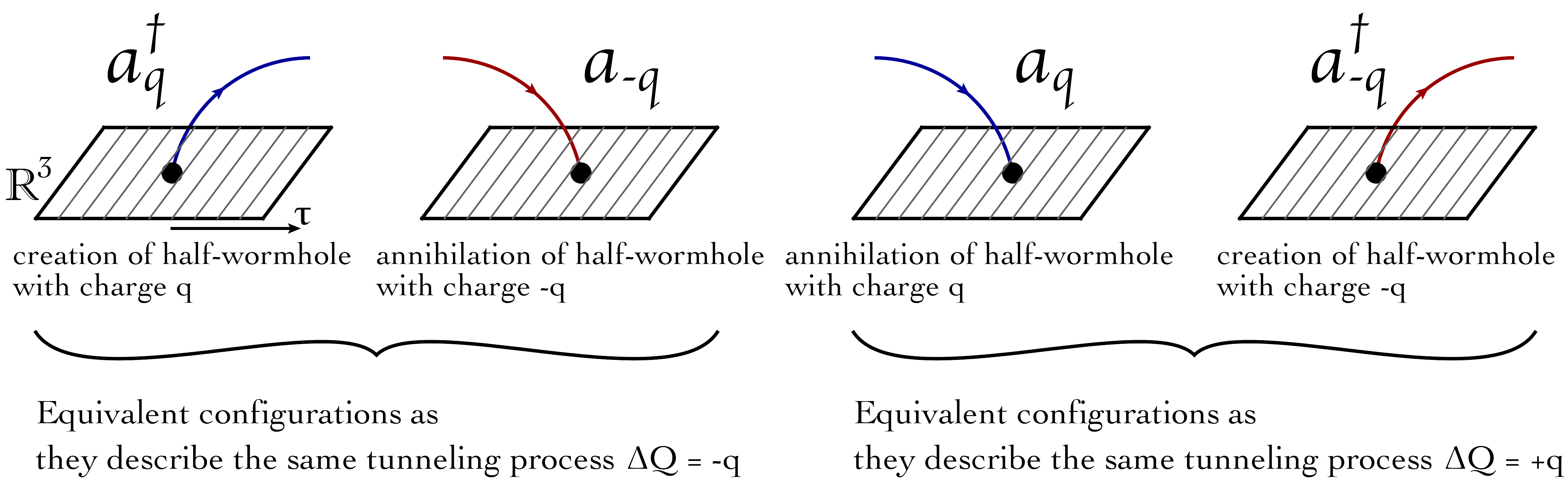
$$S = \int d^4x\sqrt{g}\sum_q K_q e^{-S}\left[(a_q^\dagger + a_{-q})\mathcal{O}_{-q} + (a_{-q}^\dagger + a_q)\mathcal{O}_q(x)\right]$$

S. J. Rey, Phys. Rev. D **39**, 3185 (1989).

$\mathcal{O}_q(x)$: an operator having charge *q*.

$$\mathcal{O}_q(x) \to e^{iq\alpha}\mathcal{O}_q(x) \qquad \theta \to \theta + \alpha$$

$$\mathcal{O}_q(x) = e^{iq\theta}\mathcal{O}_S(x)$$



Taken from R. Alonso, A. Urbano, JHEP **02**, 136 (2019).

Effective potential

S. J. Rey, Phys. Rev. D **39**, 3185 (1989).

Now define

$$C_q \equiv a_q^\dagger + a_{-q} \qquad C_q^\dagger \equiv a_{-q}^\dagger + a_q$$

$$[C_q, C_{q'}] = [C_q^\dagger, C_{q'}^\dagger] = [C_q, C_{q'}^\dagger] = 0$$

and the "coherent state":

cf.) $\mathscr{T}\,|\theta\rangle = e^{i\theta}|\theta\rangle$

$$C_q|\alpha\rangle = \alpha_q e^{i\delta_q}|\alpha\rangle \qquad C_q^\dagger|\alpha\rangle = \alpha_q e^{-i\delta_q}|\alpha\rangle$$

Tunneling transitions bring the system in the coherent state.

$$\Rightarrow \quad S = \int d^4x\sqrt{g}\sum_q 2K_q e^{-S}\alpha_q \mathcal{O}_S \cos(q\theta + \delta_q)$$

$\mathscr{O}_S$: arbitrary singlet operators