Festina Lente bound
on Higgs vacuum structure and inflation

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Festina Lente

“make haste slowly”

- ***Festina lente*** (Classical Latin: [fɛsˈtiː.naː ˈlɛn.teː]) is a classical adage (俗語) and oxymoron(撞着語法) meaning “***make haste slowly***” ***-wikipedia***
- ***If tasks are rushed too quickly then mistakes are made and good long-term results are not achieved.***
- It has been adopted as a motto, particularly by the emperor Augustus.

'Festina Lente bound'

suggested in two papers

#1

Festina Lente: EFT Constraints from Charged Black Hole Evaporation in de Sitter

Miguel Montero (Leuven U. and Harvard U.), Thomas Van Riet (Leuven U.), Gerben Venken (Leuven U.)
Oct 3, 2019

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#2

The FL bound and its phenomenological implications

Miguel Montero (Harvard U.), Cumrun Vafa (Harvard U.), Thomas Van Riet (Leuven U. and Uppsala U.), Gerben Venken (Heidelberg U.)
Jun 14, 2021

45 pages
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#1

Festina Lente: EFT Constraints from Charged Black Hole Evaporation in de Sitter

Miguel Montero(Leuven U. and Harvard U.), Thomas Van Riet(Leuven U.), Gerben Venken(Leuven U.). 1910.01648

In the Swampland philosophy of constraining EFTs from black hole mechanics we study charged black hole evaporation in de Sitter space. We establish how the black hole mass and charge change over time due to both Hawking radiation and Schwinger pair production as a function of the masses and charges of the elementary particles in the theory. **We find a lower bound on the mass of charged particles by demanding that large charged black holes evaporate back to empty de Sitter space**, in accordance with the thermal picture of the de Sitter static patch. This bound is satisfied by the charged spectrum of the Standard Model. …. Enforcing the thermal picture also leads to a heuristic remnant argument for the Weak Gravity Conjecture in de Sitter space, where the usual kinematic arguments do not work. We also comment on a possible relation between WGC and universal bounds on equilibration times. **All in all, charged black holes in de Sitter should make haste to evaporate, but they should not rush it.**

#2

The FL bound and its phenomenological implications

Miguel Montero(Leuven U. and Harvard U.), **Cumrun Vafa(Harvard U.)**, Thomas Van Riet(Leuven U.), Gerben Venken(Leuven U.). 2106.07650

Demanding that charged Nariai black holes in (quasi-)de Sitter space decay without becoming super-extremal implies a lower bound on the masses of charged particles, known as the Festina Lente (FL) bound. In this paper we fix the $O(1)$ constant in the bound and elucidate various aspects of it, as well as extensions to $d > 4$ and to situations with scalar potentials and dilatonic couplings. **We also discuss phenomenological implications of FL including an explanation of why the Higgs potential cannot have a local minimum at the origin, thus explaining why the weak force must be broken.** ….(brane-anti brane, throat and warped space …)

Charged BH in dS space

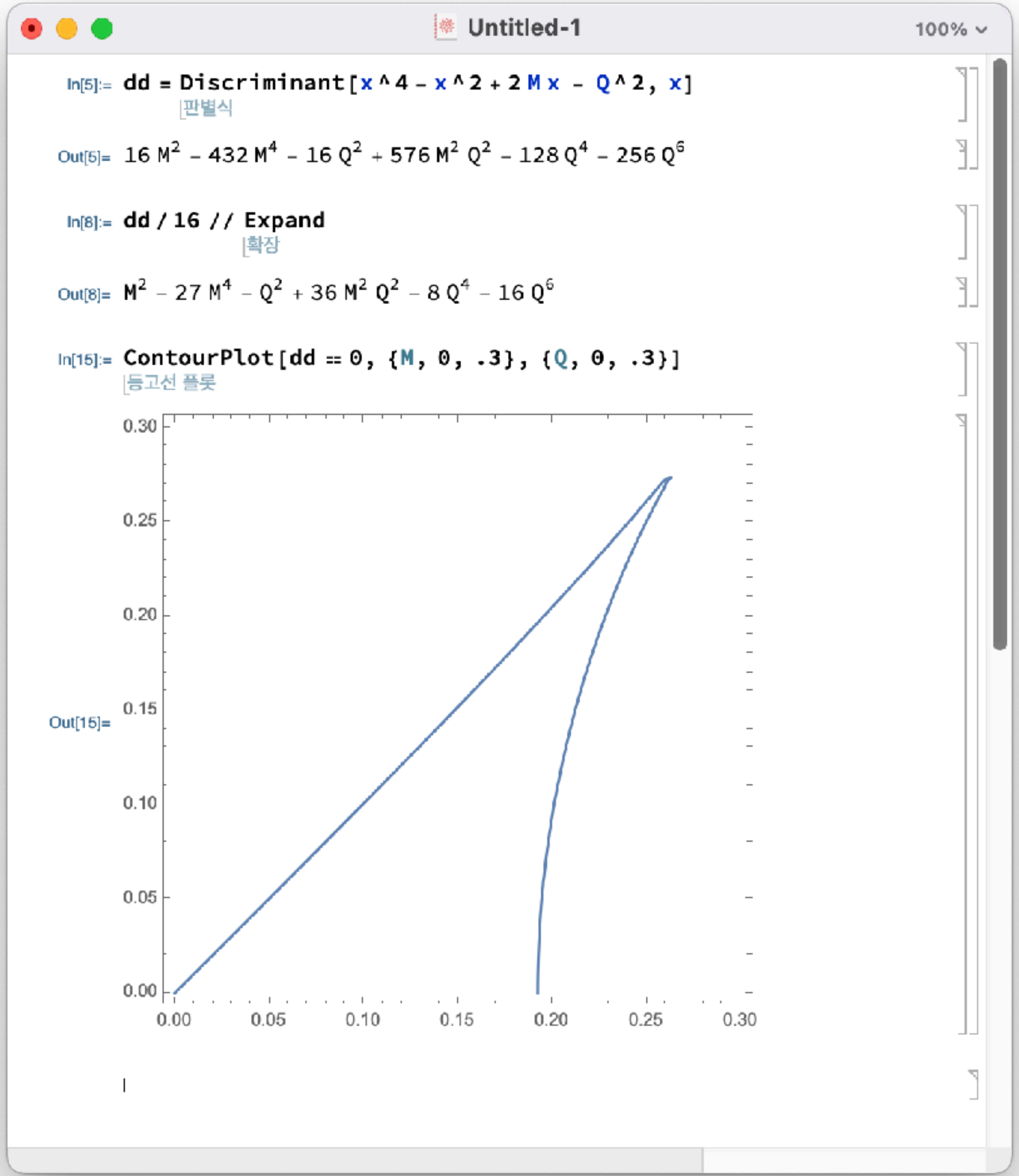
Reissner-Nordstrom-de Sitter black holes $(-,+,+,+)$

- BH solution in asymptotically dS space $\Lambda_{\rm cc} = \dfrac{3}{\ell_{\rm dS}^2} > 0$ with a gauged $U(1)$
- $S = \displaystyle\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(-R+2\Lambda_{\rm cc})+\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}\right]$
- $ds^2_{\rm RN-dS} = -U(r)dt^2 + \dfrac{dr^2}{U(r)} + r^2 d\Omega^2$
- Lapse function : $U(r) = 1 - \dfrac{2GM_r}{r} + \dfrac{G(gQ_r)^2}{4\pi r^2} - \dfrac{r^2}{\ell_{\rm dS}^2}$ with $r \in (0,\ell_{\rm dS})$
- Event horizon set at $U(r)=0$ (cf) Schwarzschild BH with $\ell_{\rm dS}\to\infty, Q_r \to 0$

Solving $0 = U(r) = 1 - \frac{2GM_r}{r} + \frac{G(gQ_r)^2}{4\pi r^2} - \frac{r^2}{\ell_{\mathrm{dS}}^2}$

setting $\ell_{\mathrm{dS}} = 1, Q = \frac{\sqrt{G}(gQ_r)}{\sqrt{4\pi}\ell_{\mathrm{dS}}}, M = \frac{GM_r}{\ell_{\mathrm{dS}}^2}$

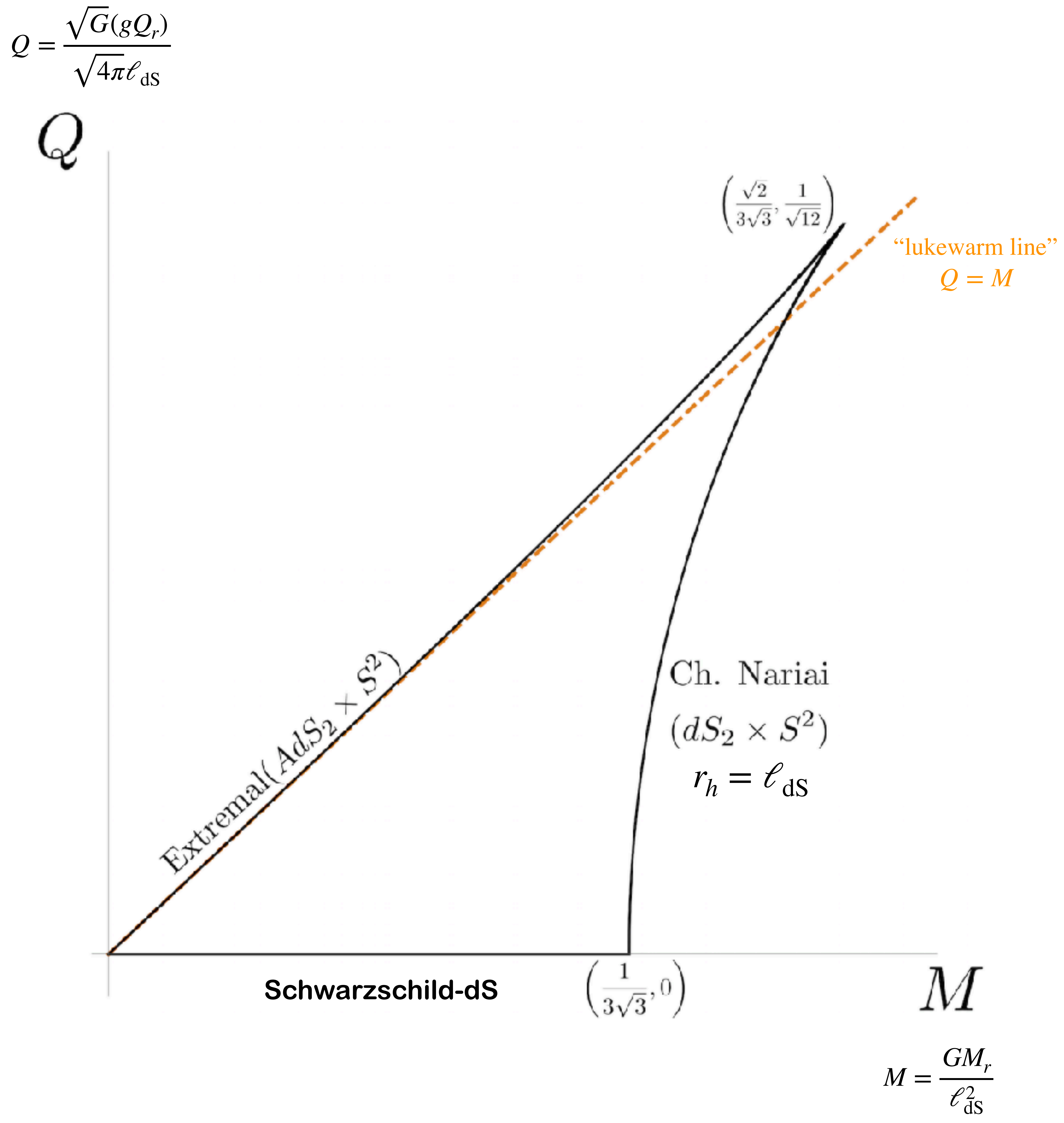
- $0 = -r^2U(r) = r^4 - r^2 + 2Mr - Q^2$ solvable when $\Delta \geq 0$
- discriminant for quartic eq. $\Delta = M^2 - Q^2 - 27M^4 + 36M^2Q^2 - 8Q^4 - 16Q^6$
- $\Delta = 0$ defines the physical domain in (Q, M) space.



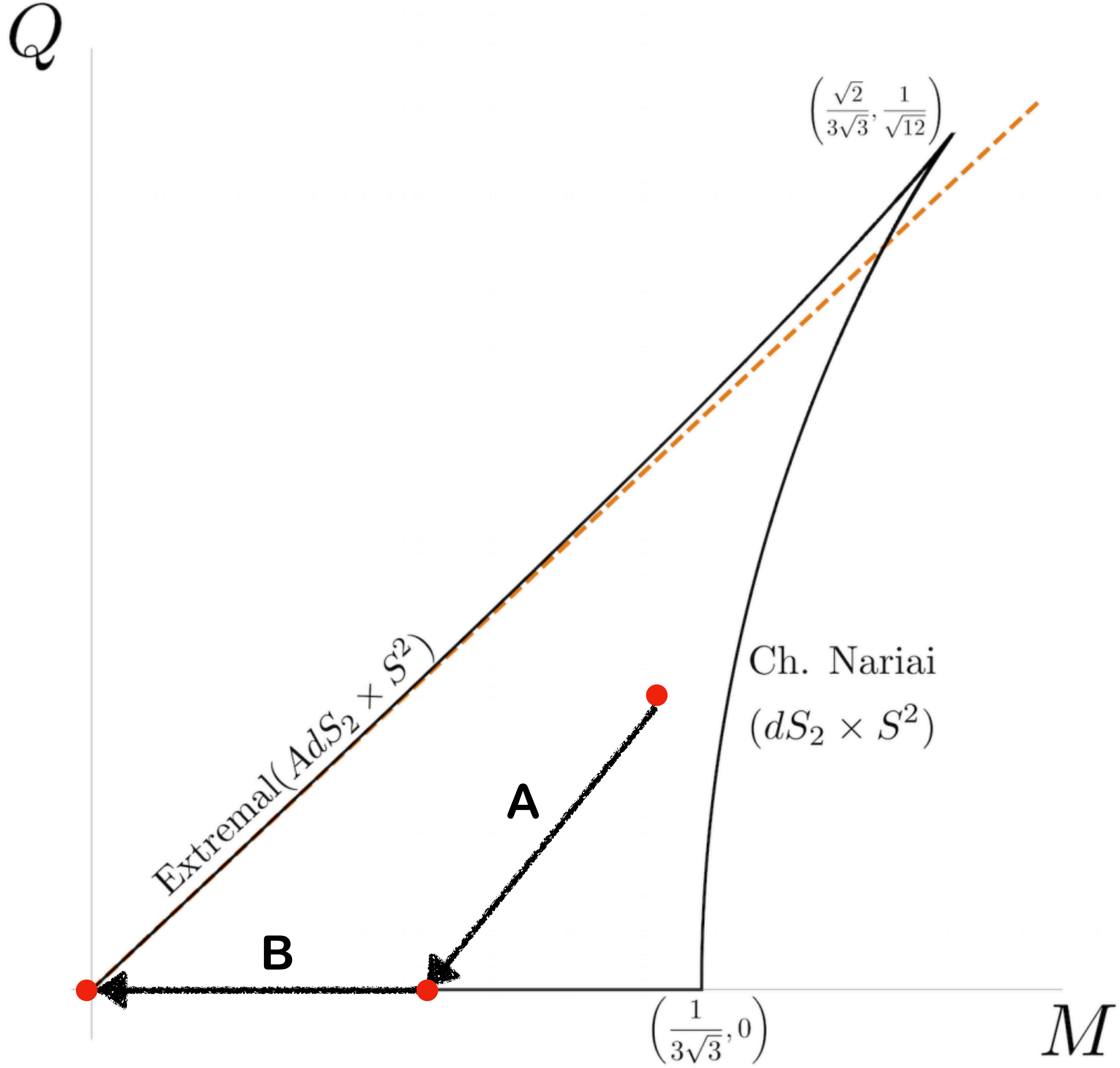
Phase diagram

'Shark fin'

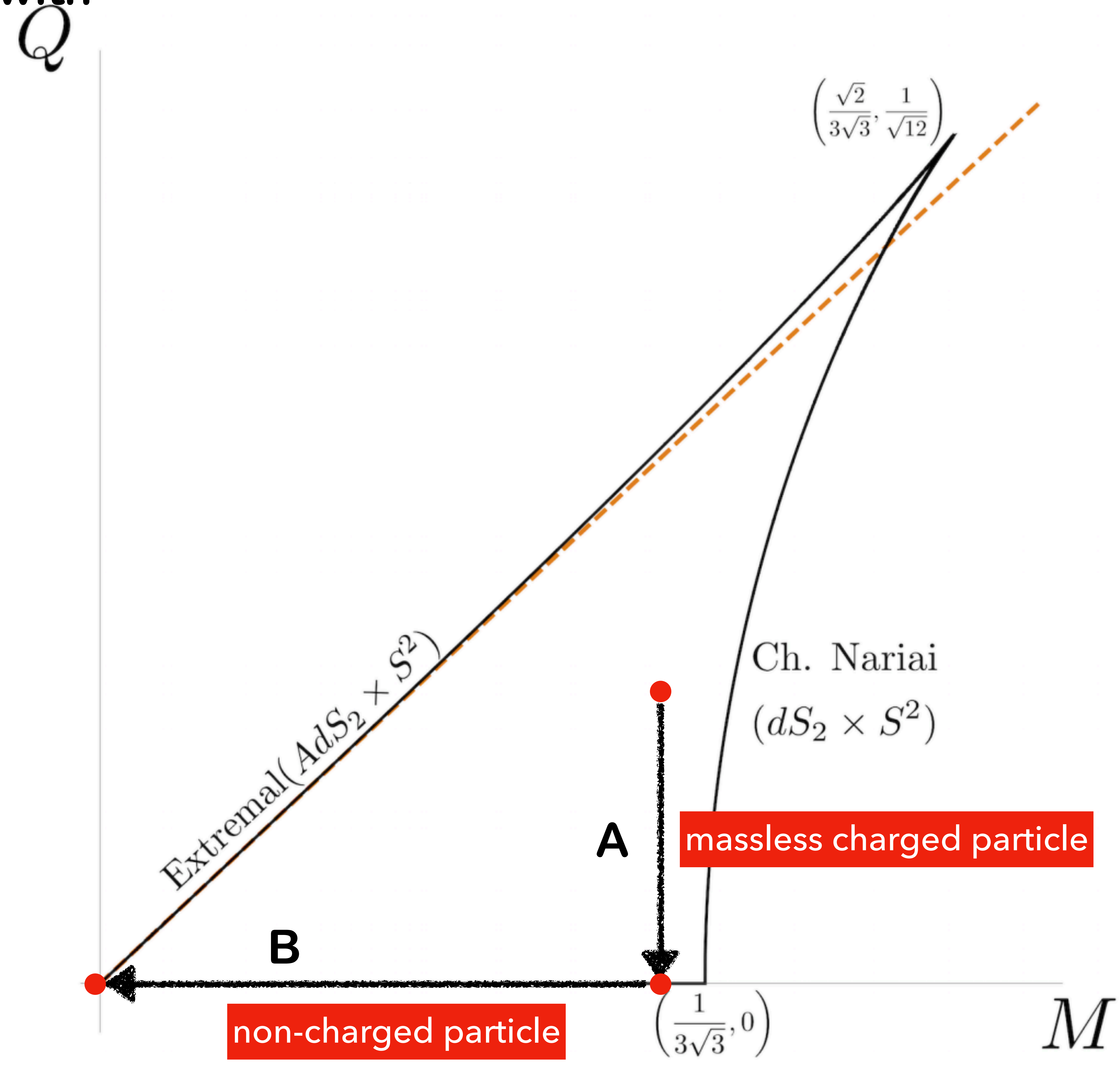
- Outside of Shark fin is unphysical (super-extremal)
- Extremal branch has $AdS_2 \times S^2$ topology
- Charged Nariai branch has $dS_2 \times S^2$ near horizon geometry
- BH should remain inside during its evolution till its decay



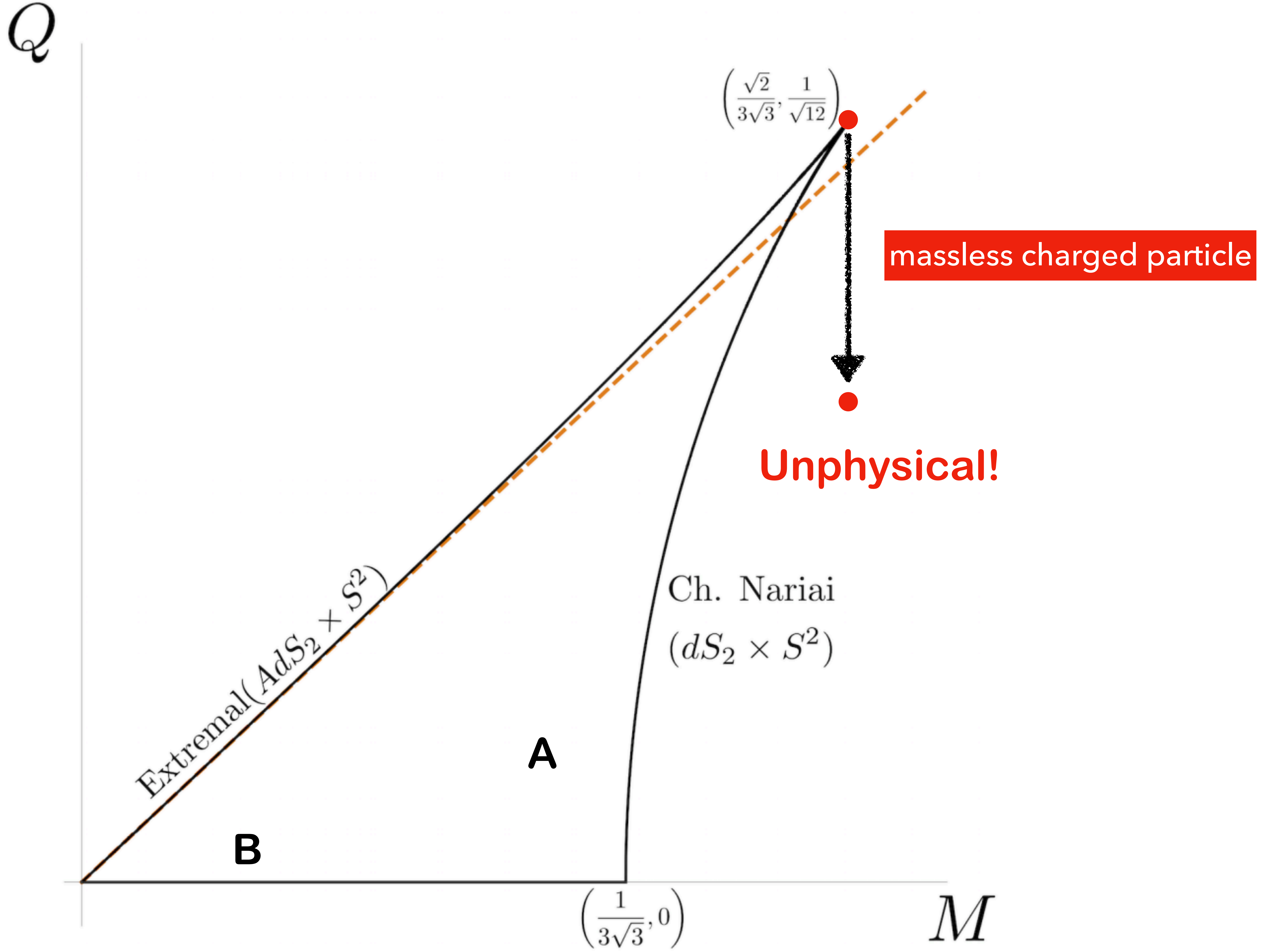
(EX) Evaporation



(EX2) Evaporation with $m = 0, q \neq 0$)



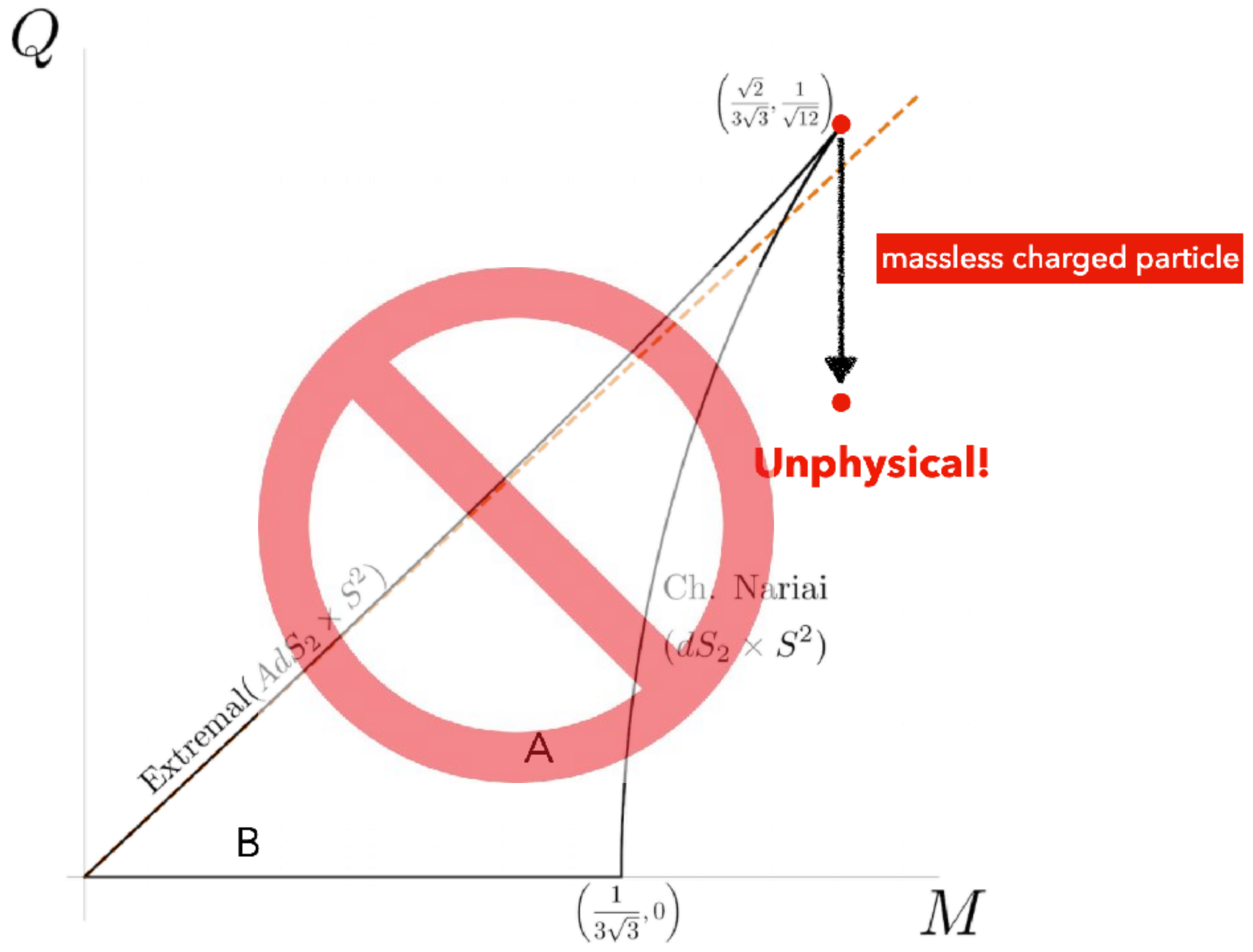
(EX3) Unphysical evaporation with $m = 0, q \neq 0$)



FL bound

1910.01648 & 2106.07650

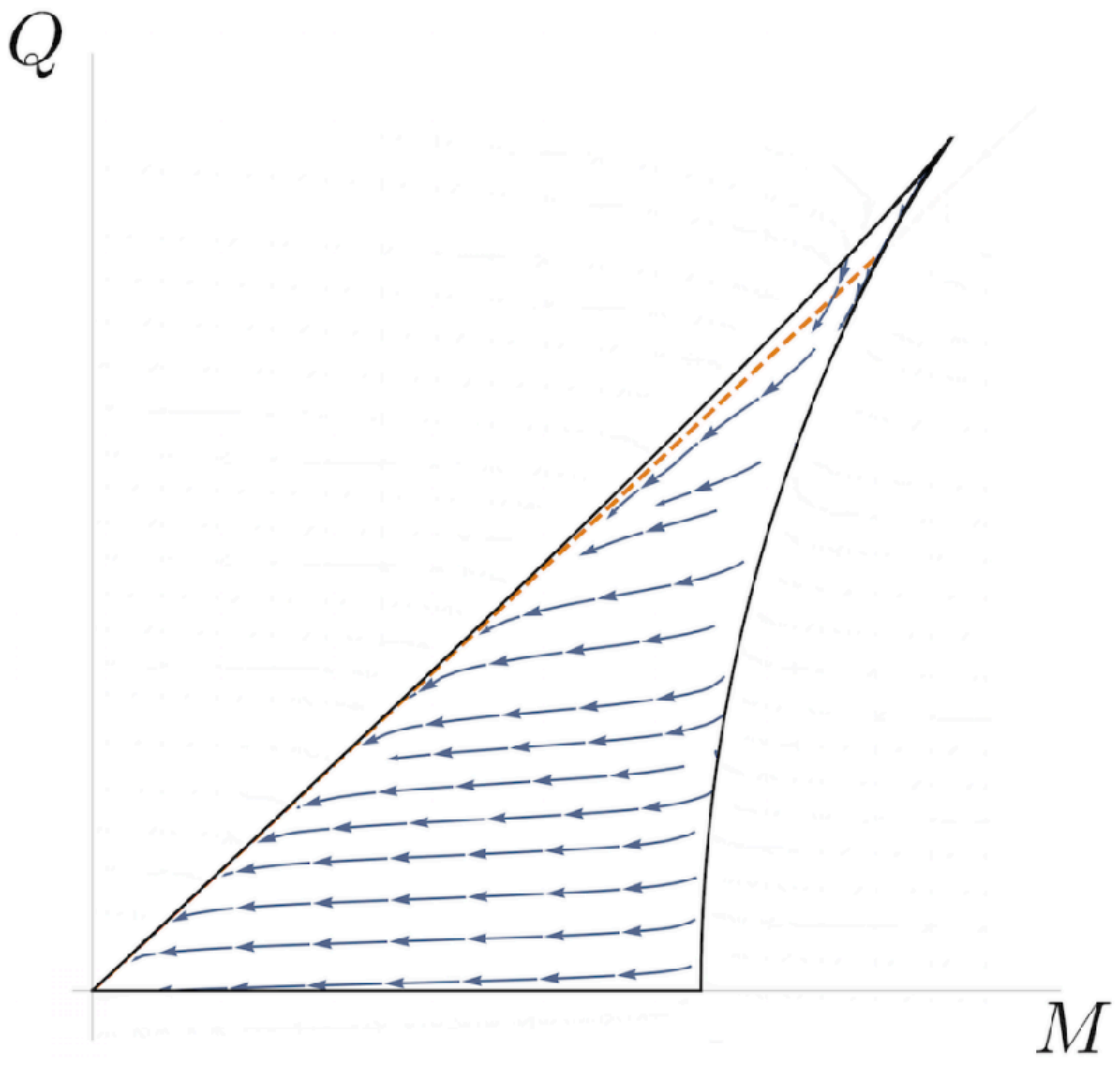
- To forbid unphysical evolution of BH, there should be a lower bound on the mass of a charged particle.
- **BH decay by Hawking radiation of energy & charge. It is fast but not too fast!**



FL bound

1910.01648 & 2106.07650

- **There should not exist a charged, massless particle.**
- **More generally, there should be a lower bound on the mass of a charged particle for the given BH solution.**



FL bound : $m^4 > 8\pi\alpha q^2 V$

(paper #1)

- m : mass of a charged particle under unbroken $U(1)$ gauge symmetry
- $\alpha = g^2/4\pi$: fine-structure constant of $U(1)$
- q : charge of the particle in unit charge $Q = qe$
- V : scalar potential energy (or CC) of dS background
- *All* charged particles should be heavier than the critical mass given by dS vacuum energy.

Applicability of FL bound

- For sure, dS vacuum at min of potential. (stable background)
- more generally, pseudo dS with slow-roll potential (meta stable) satisfying a short lifetime of blackhole : $\tau_{\rm BH} \ll \tau_{\rm Universe}$
- More precisely, the background geometry after the charged BH production is required to be deformed close to that of the Nariai BH, $dS_2 \times S^2$, which undoubtedly includes the nearly constant cosmological horizon case.
- $\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$ with $V = \frac{\lambda_{eff}(\phi)}{4}\phi^4 \;\; \rightarrow \epsilon_V = \frac{M_P^2}{2}\left(\frac{\lambda'_{\rm eff}}{\lambda_{\rm eff}} + \frac{4}{\phi}\right)^2 = \frac{8M_P^2}{\phi^2}\left(1 + \frac{\beta_\lambda^{\rm eff}}{4\lambda_{\rm eff}}\right)^2$
- For Narai bh with $g\sqrt{V}$ being electric field ($E_{\rm Narai}$), and we request $\epsilon_V \ll e^{-m^2/qE_{\rm Narai}} < 1$

Phenomenological implications

at EW vacuum

- Within the SM, the lightest charged particle is electron with $m_e = 0.511\text{MeV}, q = 1 (Q = e)$ for $U(1)_{\text{em}}$
- At current universe, $V = \dfrac{\Lambda_{cc}}{8\pi G} = \rho_{vac}$ from cosmological measurement:
  $\Lambda_{cc} = 8\pi G \rho_{vac} = 3(H_0)^2 \Omega_\Lambda = 2.8 \times 10^{-122} M_P^2$ (measured by Planck)
- FL bound ($8\pi\alpha V \ll m_e^4$) easily satisfied!
- This is due to the fact that we are in a broken phase $\langle H \rangle = 246\ \text{GeV}$, $m_e = \dfrac{y_e}{\sqrt{2}} \langle H \rangle$.
- FL bound tells us that $\langle H \rangle \geq \left[\dfrac{32\pi\alpha V}{y_e^4} \right]^{1/4}$ or **EW symmetry should be broken!** (surprise?)
- Note) cosmological constant problem being slightly relieved in FL region

The Higgs in the SM

Goldstone

$$H \sim \begin{pmatrix} G^+ \\ (v + h + G^0)/\sqrt{2} \end{pmatrix}$$

vev ↑ Goldstone

physical Higgs

(3,2,1/2)

of $SU(3) \times SU(2) \times U(1)$

$$v = \sqrt{\frac{1}{\sqrt{2} G_F}} = 246.22 \text{ GeV}$$

The Higgs potential energy

The most general, gauge invariant, renormalizable potential

$$V_{\text{Higgs}} = \boxed{\lambda}(|H|^2 - \boxed{v^2}/2)^2$$

Only two free parameters!

unitary gauge

$$|H|^2 = \frac{(v+h)^2}{2}$$

$$= \frac{\lambda}{4}h^4 + \lambda v h^3 + \lambda v^2 h^2$$

λ_{hhhh} λ_{hhh} $m_h^2/2$

These terms are correlated in the SM

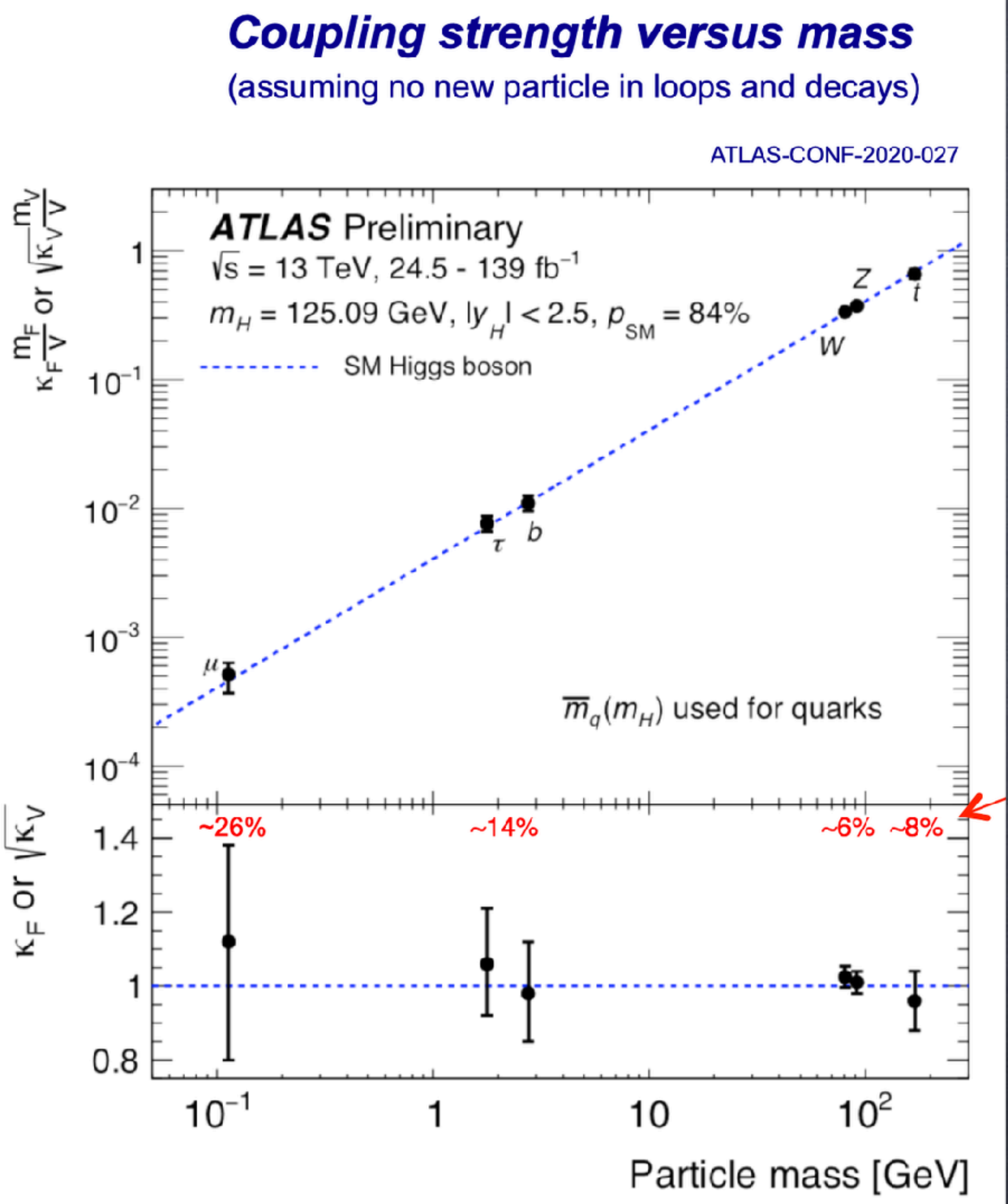
The roles of the Higgs

-Masses of elementary particles : experimentally confirmed
-EWSB : experimentally confirmed
-Cosmological inflation : theoretically suggested
(not this talk)

Beautifully confirmed by the LHC!

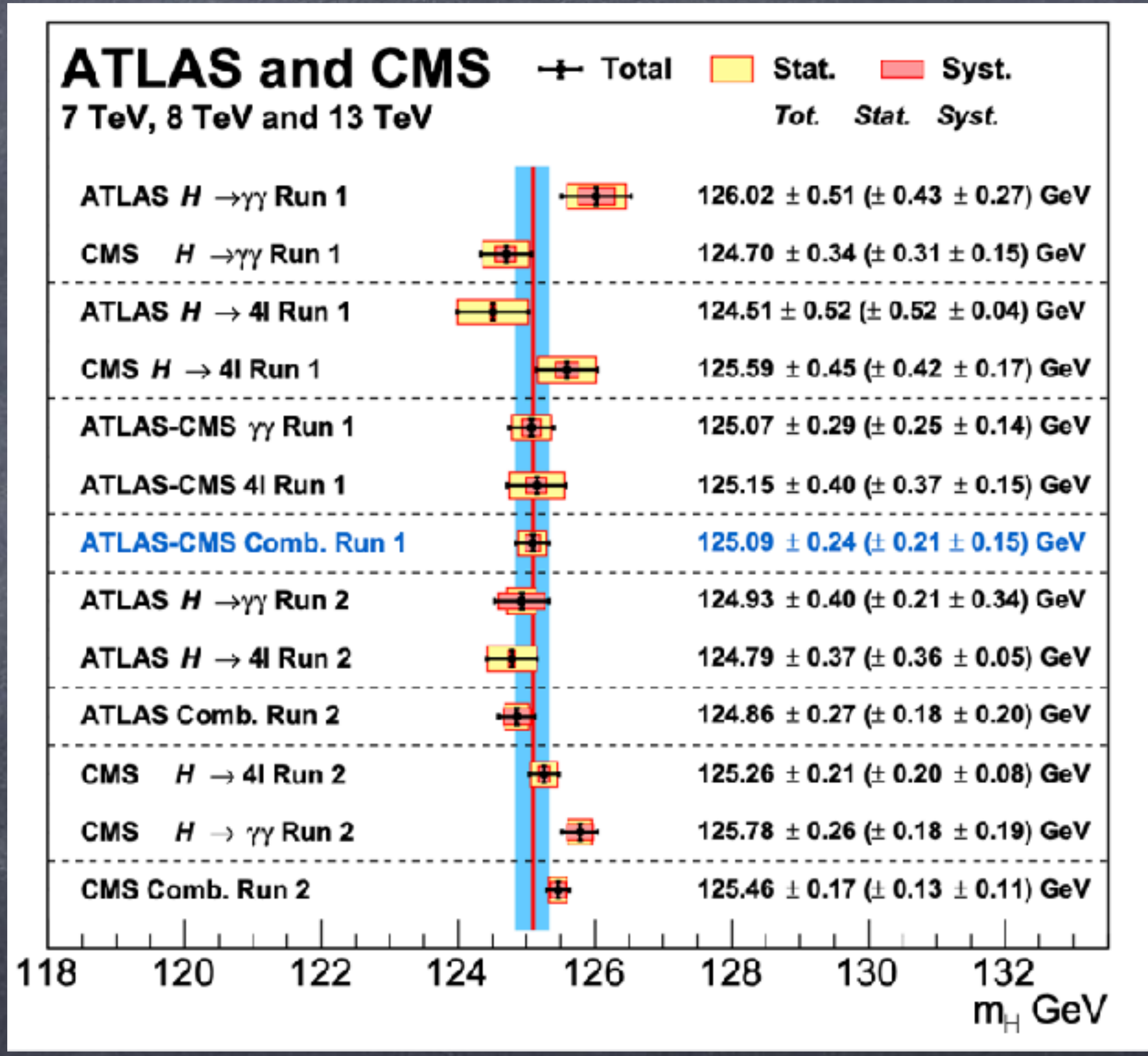
a linear relation

$$m_\psi, m_Z \propto \langle H \rangle$$



Higgs self-coupling

$$\lambda^{\mathrm{SM}} = \frac{m_h^2}{2v^2} = \frac{125^2}{2 \times 246^2} \approx \frac{1}{8}$$



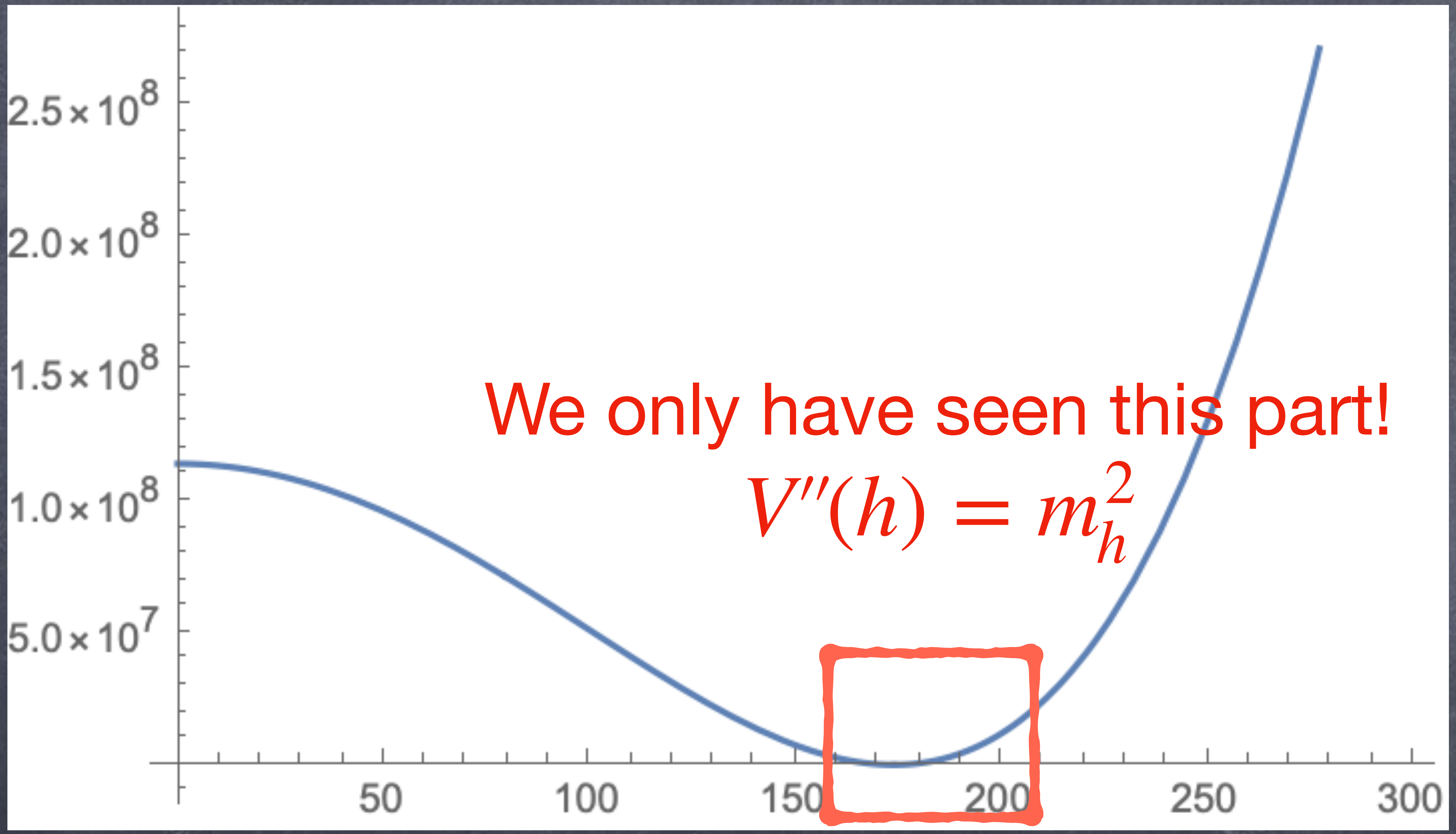
Higgs mass

The SM Higgs potential

$$V_{\text{Higgs}} = \frac{1}{32}h^4 + \frac{246\ \text{GeV}}{8}h^3 + \frac{1}{2}(125\ \text{GeV})^2 h^2$$

Predicted Predicted only this term is confirmed by LHC

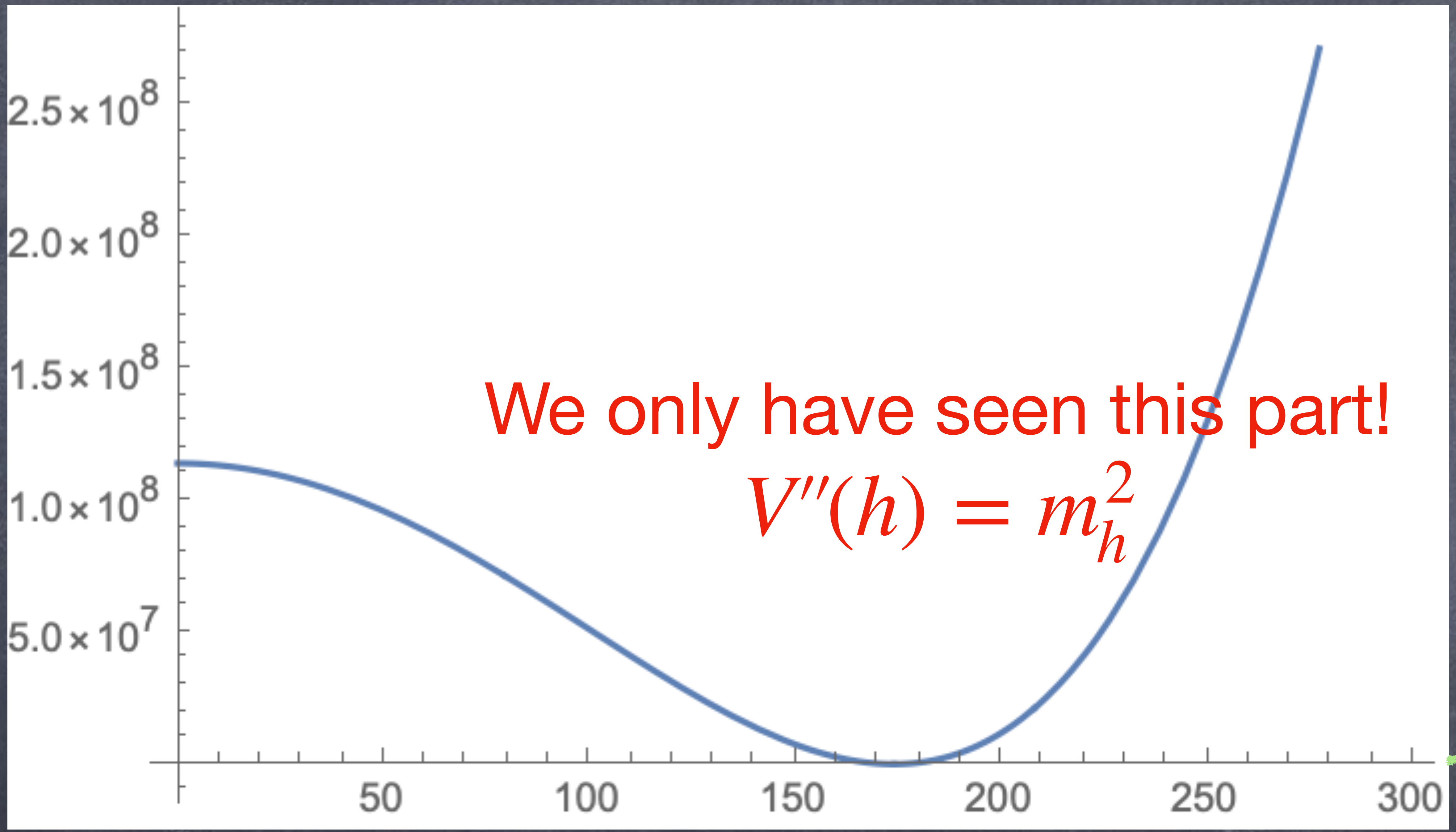
Higgs potential near EW vacuum



The SM prediction

$$V(H) = \lambda(|H|^2 - v^2/2)^2$$

Higgs potential at high scale



The SM prediction

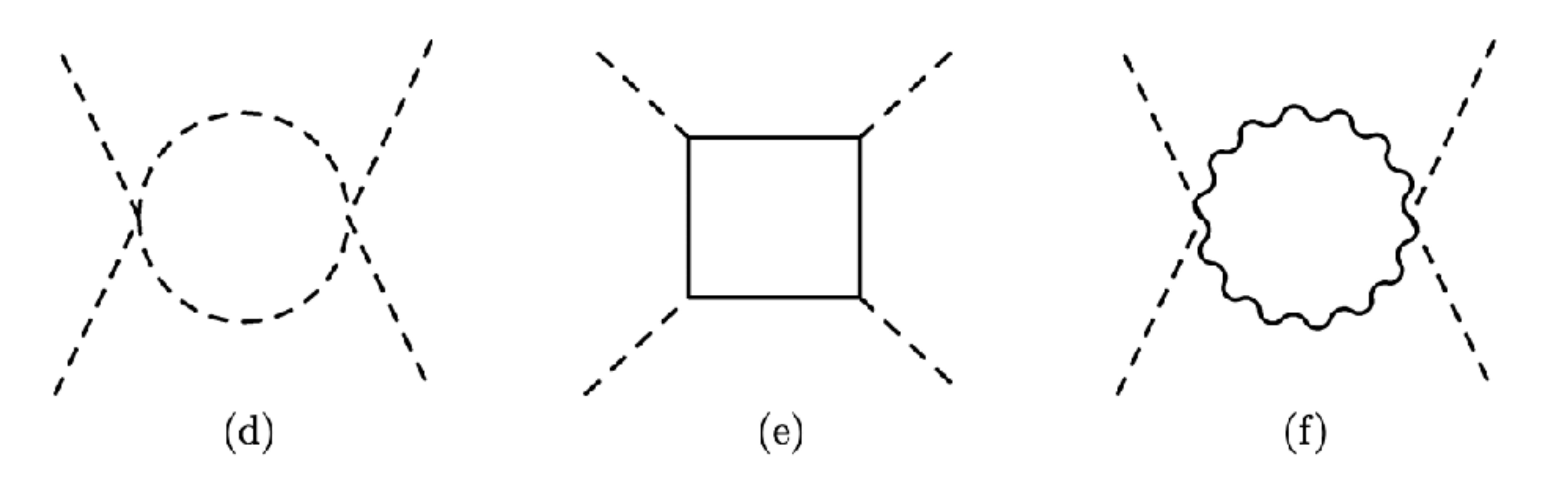
$$V(H) = \lambda(|H|^2 - v^2/2)^2$$

$$\boxed{h \gg v_{\rm EW}}$$

Q. what's happening over here?

Quantum effects on λ_{hhhh}

Simone, Hertzberg, Wilczek (PLB 2009), Hamada, Kawai, Oda, SCP (PRL 2014)



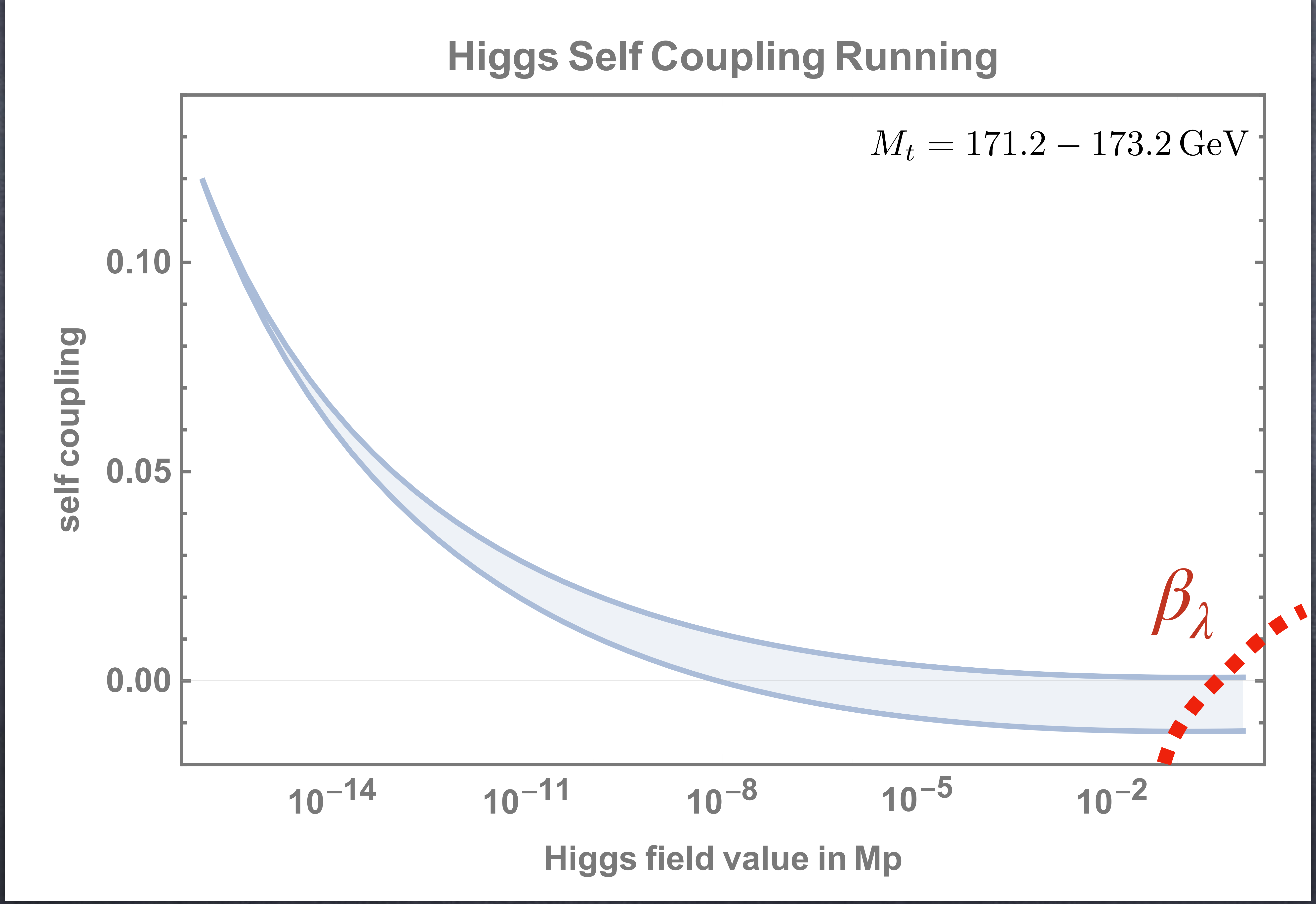
$$\frac{d\lambda}{d\log\mu} =$$

$$\begin{aligned}
\beta_\lambda \;=\; & \frac{1}{(4\pi)^2}\left[24s^2\lambda^2 \boxed{-6y_t^4} + \frac{3}{8}\left(2g^4 + \left(g^2+g'^2\right)^2\right) + \left(-9g^2 - 3g'^2 + 12y_t^2\right)\lambda\right] \\
+\; & \frac{1}{(4\pi)^4}\left[\frac{1}{48}\left(915g^6 - 289g^4g'^2 - 559g^2g'^4 - 379g'^6\right) + 30sy_t^6 - y_t^4\left(\frac{8g'^2}{3} + 32g_s^2 + 3s\lambda\right)\right. \\
+\; & \lambda\left(-\frac{73}{8}g^4 + \frac{39}{4}g^2g'^2 + \frac{629}{24}sg'^4 + 108s^2g^2\lambda + 36s^2g'^2\lambda - 312s^4\lambda^2\right) \\
+\; & \left. y_t^2\left(-\frac{9}{4}g^4 + \frac{21}{2}g^2g'^2 - \frac{19}{4}g'^4 + \lambda\left(\frac{45}{2}g^2 + \frac{85}{6}g'^2 + 80g_s^2 - 144s^2\lambda\right)\right)\right]. \qquad (33)
\end{aligned}$$

< 0

==> weaker at higher energies!!

RG running of λ



Higgs Criticality!

$$\lambda \approx 0 \approx \lambda'$$

Hamada, Kawai, Oda, SCP (PRL 2014)

More on Higgs potential

S.M.Lee, D.Y.Cheong, S.C.Hyun, SCP, M.-S.Seo

arXiv 2111.04010

- RG running of λ, and higher order operators

$$V_{\rm eff}(h) = \Lambda_{\rm DE} + \frac{\lambda(h)}{4}h^4 + \frac{c_6}{\Lambda^2}h^6 + \frac{c_8}{\Lambda^4}h^8 + \cdots .$$

- 1 : the unique EW vacuum
- 2″ : inflection point $V' = 0 = V''$
- 2, 2′ : 2nd dS vacuum at UV
- 3 : 2nd vacuum with AdS

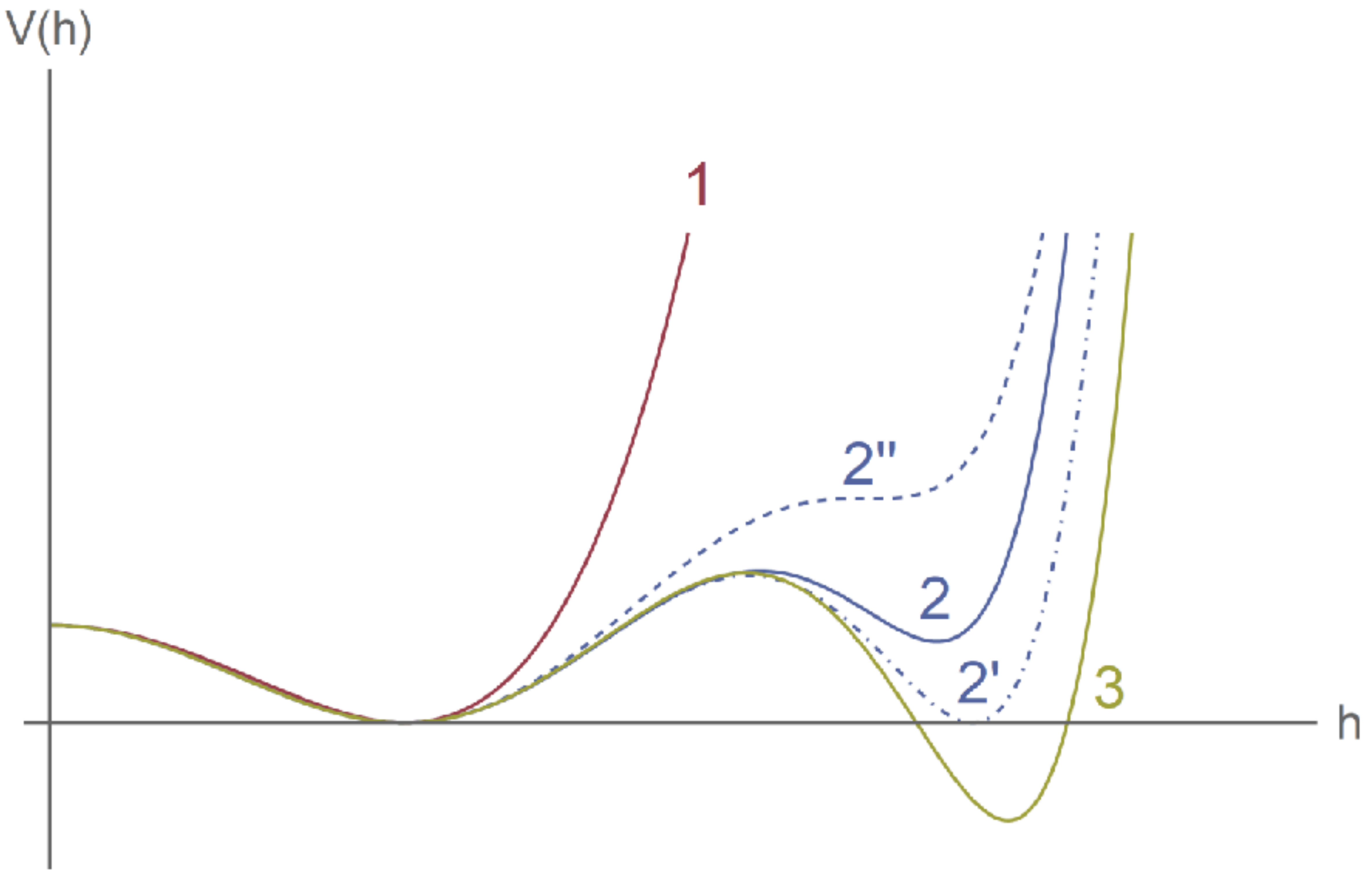


Figure 1: Schematic shape of the Higgs potential.

More on Higgs potential

S.M.Lee, D.Y.Cheong, S.C.Hyun, SCP, M.-S.Seo

arXiv 2111.04010

- 1 : the unique EW vacuum.
- consistent with FL bound at the EW vacuum with a tiny CC
- $\Lambda_{cc} = 8\pi G\rho_{vac} = 3(H_0)^2\Omega_\Lambda = 2.8 \times 10^{-122} M_P^2$
- FL bound:

$$\Lambda_{cc} \leq \frac{Gm_e^4}{\alpha} \sim 10^{-90} M_P^2$$

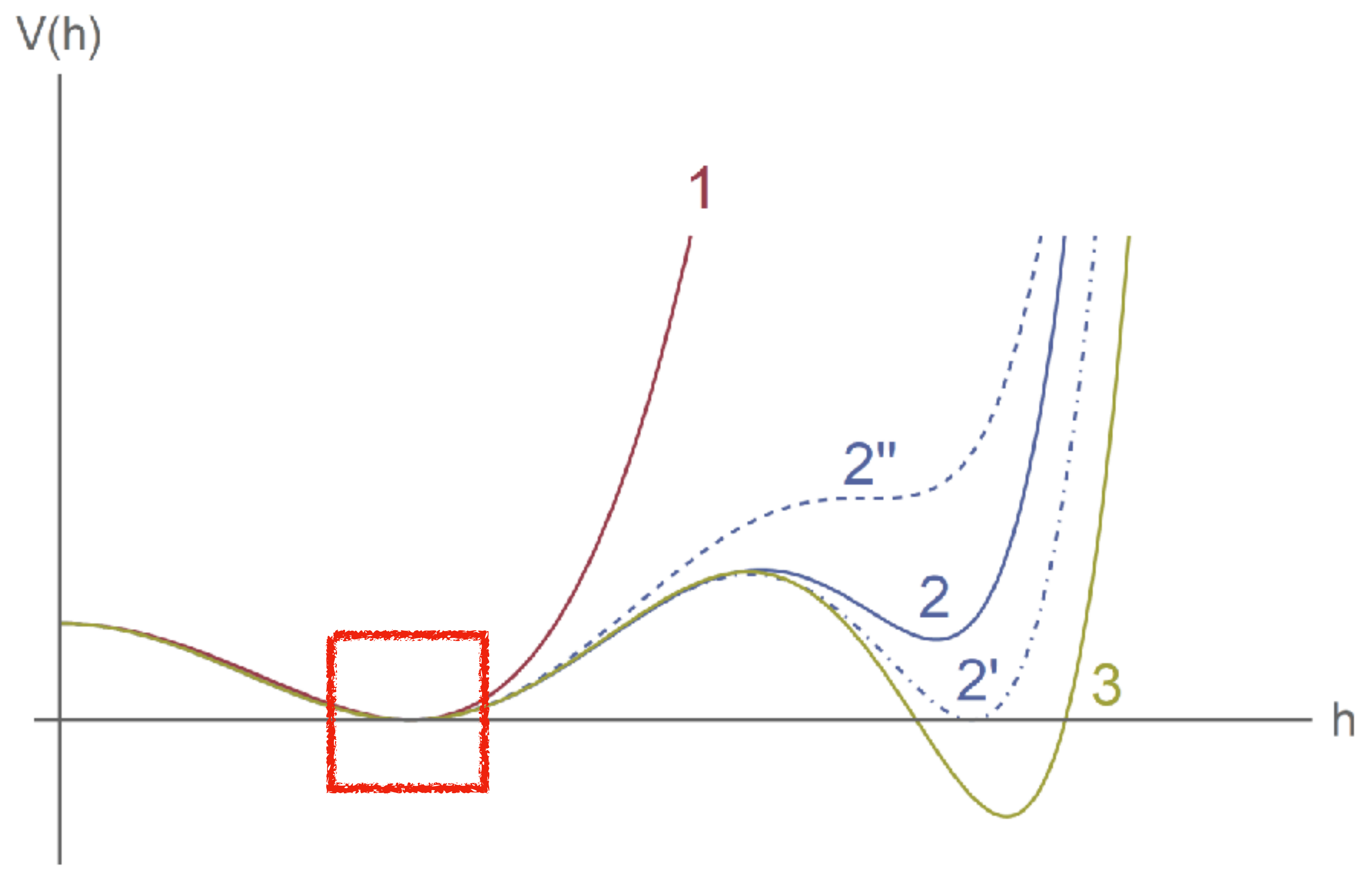


Figure 1: Schematic shape of the Higgs potential.

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- 3 : 2nd vacuum with AdS

  => FL bound not applied

  We don't exclude this possibility here.

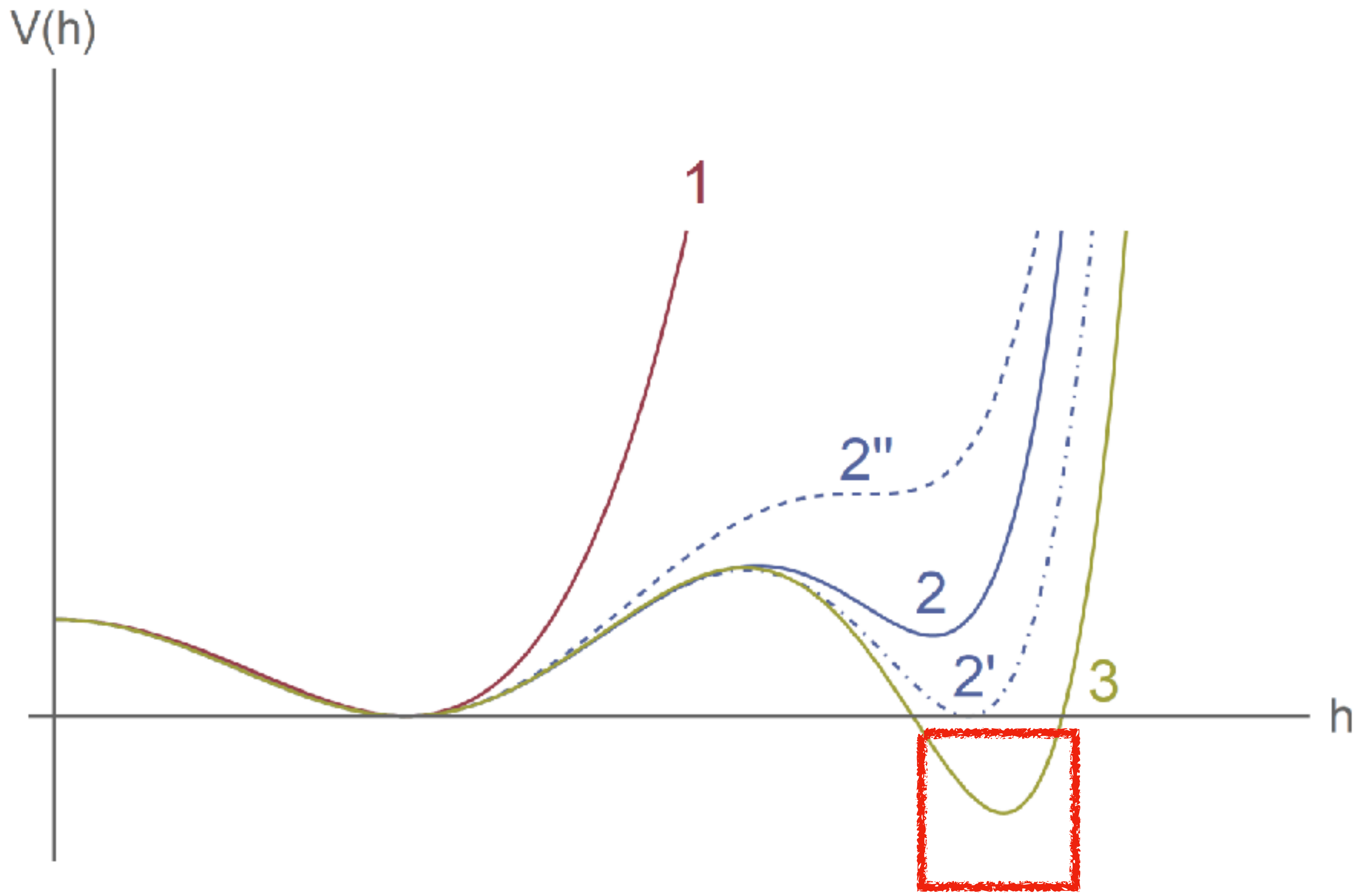

Figure 1: Schematic shape of the Higgs potential.

More on Higgs potential

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- 2″, 2, 2′ : 2nd dS vacuum at UV

$$\min_{i\in \mathrm{SM}} \frac{m_i^4}{8\pi\alpha_i} = \frac{y_e^4 v_{\mathrm{UV}}^4/4}{8\pi\alpha_{\mathrm{EM}}} \geq \frac{\lambda_{\mathrm{eff}}(v_{\mathrm{UV}})}{4} v_{\mathrm{UV}}^4$$

$$\Rightarrow \quad \lambda_{\mathrm{eff}}(v_{\mathrm{UV}}) \leq \frac{y_e^4}{8\pi\alpha_{\mathrm{EM}}} \simeq \mathcal{O}\left(10^{-22}\right).$$

- Nearly degenerate vacuum (2′) is allowed by FL bound (the potential cannot be too high!)

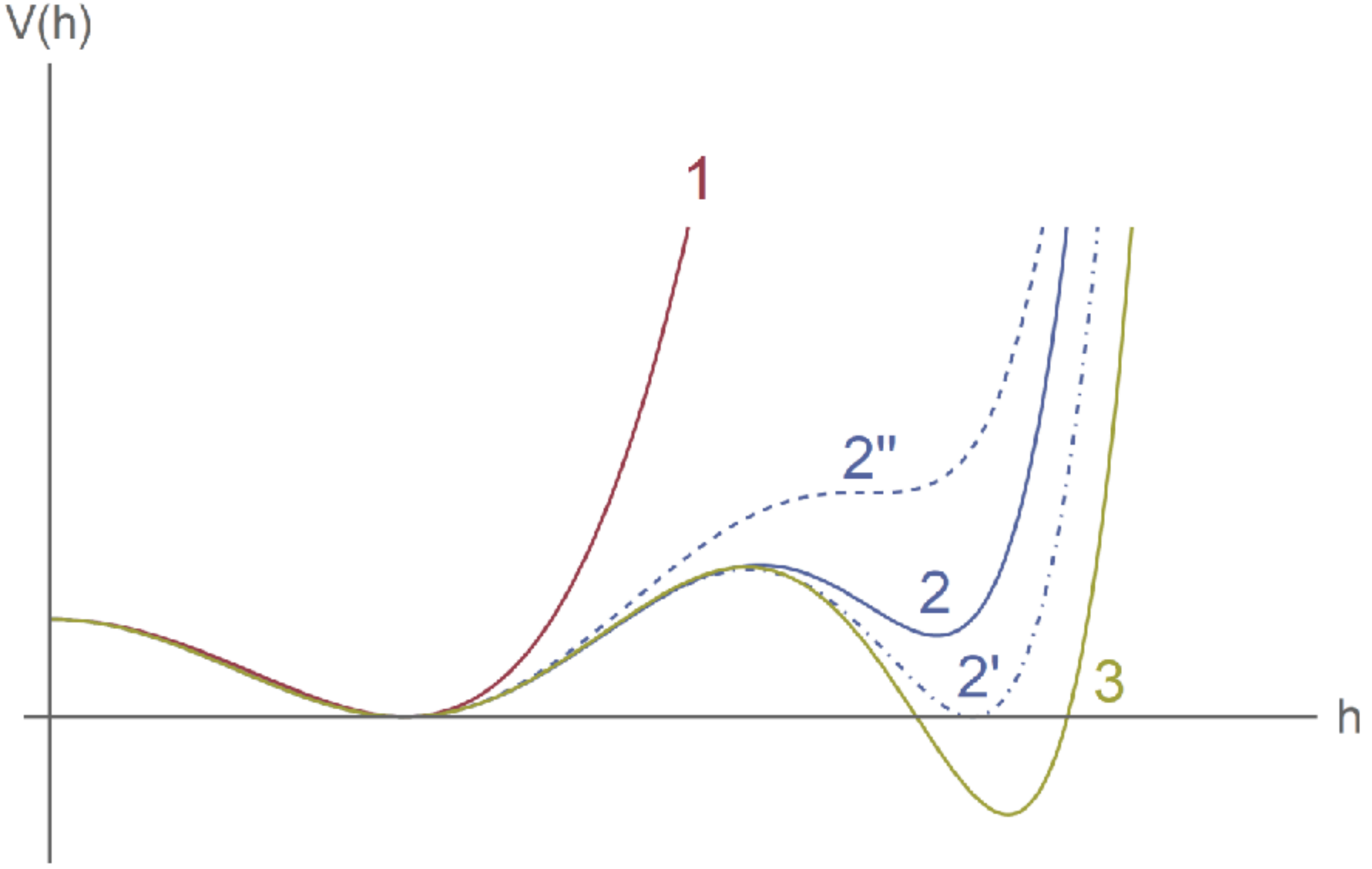
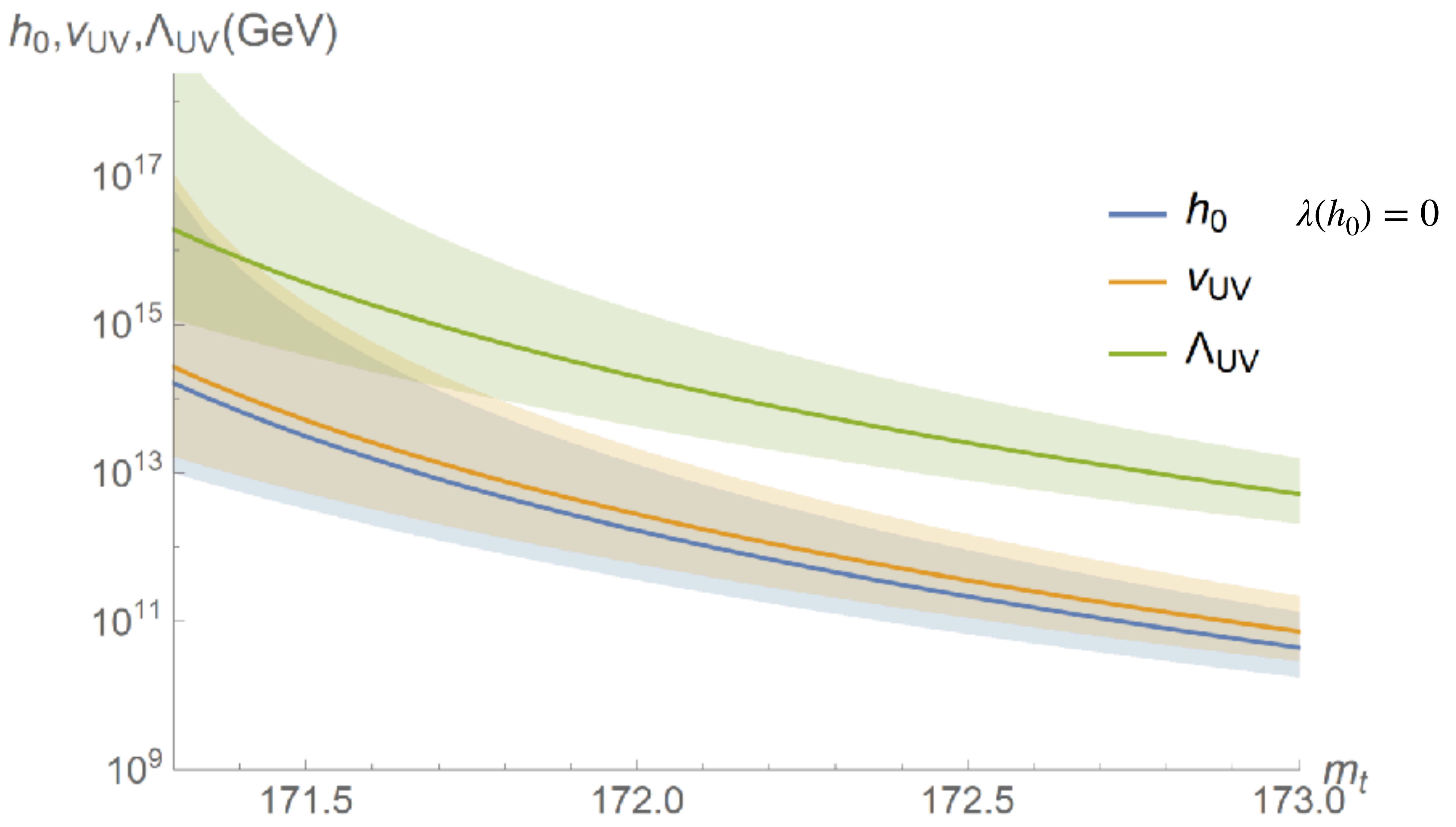


Figure 1: Schematic shape of the Higgs potential.

More on Higgs potential
at degenerate vacuum (2′)

- $V_{\rm eff} = \frac{\lambda(h)_{\rm eff}}{4}h^4 = \frac{\lambda(h)}{4}h^4 + \frac{h^6}{\Lambda} + \cdots$ with $\lambda(h) = \lambda_* - \frac{b_1}{16\pi^2}\log\frac{h}{h_*}$ near degenerate vacuum $V_{\rm eff}(v_{UV}) = 0 = V'_{\rm eff}(v_{UV})$
- $v_{UV} = h_* e^{\frac{16\pi^2\lambda_*}{b_1}+\frac{1}{2}}$, $\Lambda_{UV} = \frac{8\sqrt{2}\pi}{\sqrt{b_1}}v_{\rm UV}$
- Taking the RG running effect $\lambda(h) = -\frac{b_1(m_t)}{32\pi^2}$ (uncertainty αs = 0.1179 ± 0.0010) with respect to top mass provides the relations to $h_0, v_{UV}, \Lambda_{UV}$



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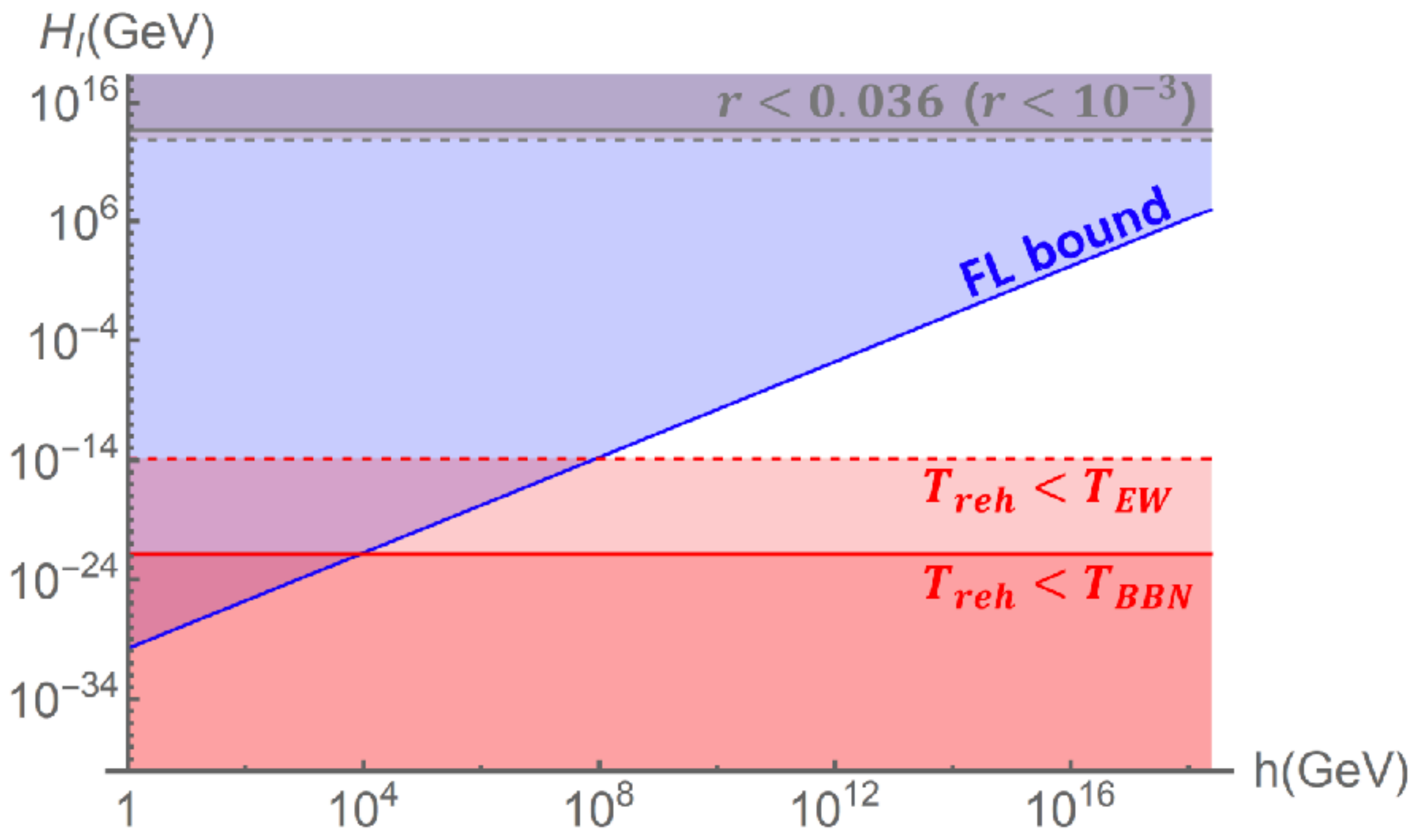
Inflation

with additional fields

- More scalars $\varphi_i, i = 1,2,3,\cdots$
- $\sum_i U_i(\varphi_i) + V(h) = 3M_P^2 H_I^2$, Hubble parameter during inflation
- FL bound $\frac{y_e^4 h^4/4}{8\pi\alpha} \geq 3M_P^2 H_I^2$ or $h \geq \left(\frac{96\pi\alpha}{y_e^4}\right)^{1/4} \sqrt{M_P H_I}$
- Higgs cannot stay at EW vacuum during inflation (whatever inflaton was!)



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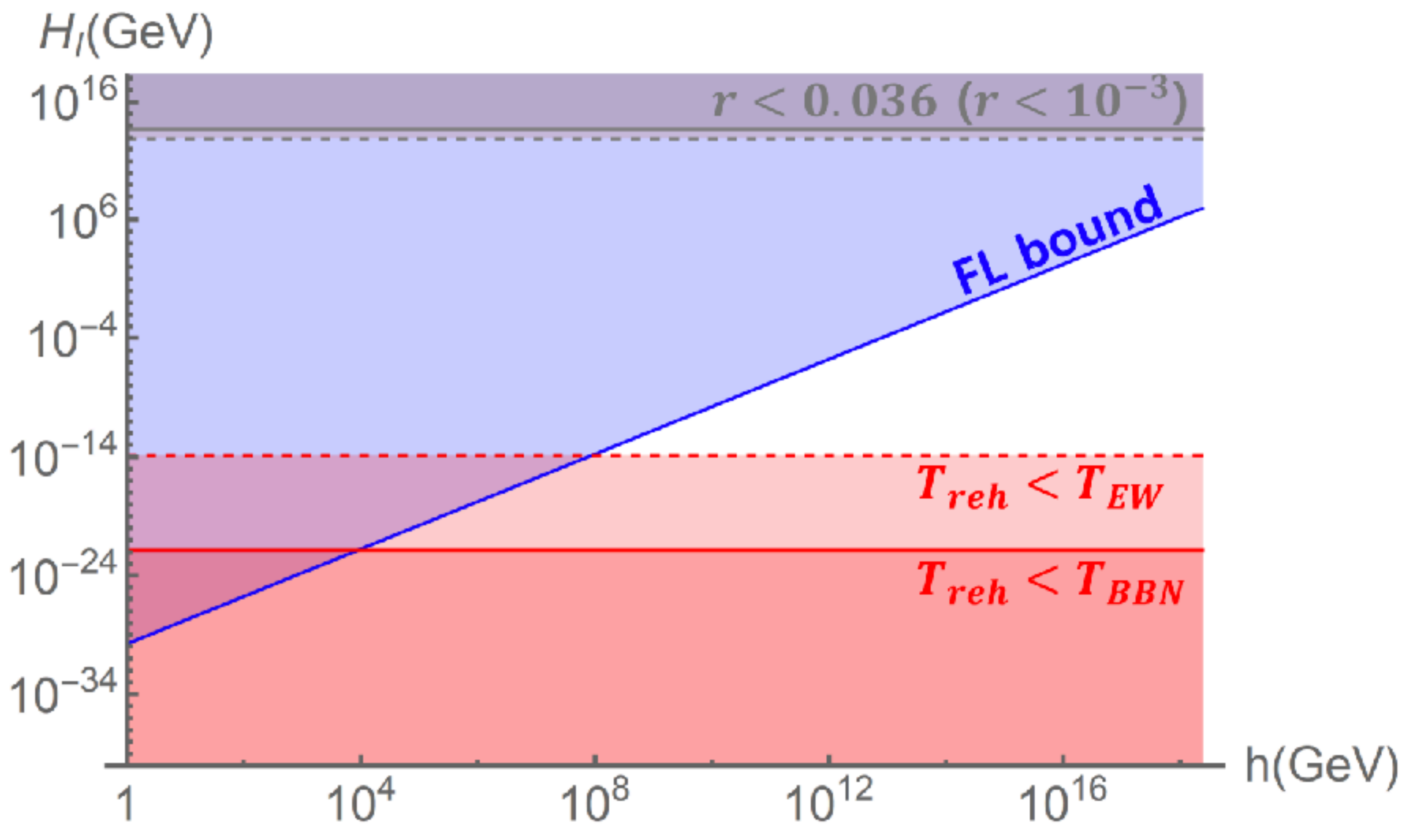
Inflation

small tensor-to-scalar ratio

- $H_I^2/M_P^2 \simeq \frac{\pi^2}{2} A_S r$

requests small tensor-to-scalar ratio

- $r \lesssim 3 \times 10^{-15} \left(\frac{10^{-2}}{\alpha_{EM}}\right)\left(\frac{2 \cdot 10^{-9}}{A_S}\right)\left(\frac{y_e}{3 \cdot 10^{-6}}\right)^4 \frac{h^4}{M_P^4}$
- Requesting reheating temperature high enough (at least BBN, or EW symmetry breaking), we learn the lower bound on H_I during inflation
- FL bound set upper bound on H_I (potential cannot be too high)
- Only, limited case is consistent with FL bound and cosmology!



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Dark gauged $U(1)_D$

- The electron is stable because it is the lightest charged particle under $U(1)_{em}$
- The dark matter is stable if it is the lightest charged particle under $U(1)_D$
- FL bound forbid too light DM:

$m_D \geq (8\pi\alpha_D q_D^2 V_{cc})^{1/4} = (8\pi\alpha_D q_D^2 \frac{\Lambda_{cc}}{8\pi G})^{1/4} = (\alpha_D q_D^2 \Lambda_{cc})^{1/4} \sim 10^{-31} M_P \sim 10^{-3}\mathrm{eV}$ with $\alpha_D \sim \alpha, q_D = 1$

- This excludes FIMP at $m_{FIMP} \sim 10^{-22}\mathrm{eV}$
- Dark radiation is another component we need to consider (which may mix with photon)

Conclusion

- FL bound is found from BH decay in dS vacuum $m_q \geq (8\pi\alpha q^2 V)^{1/4}$
- Taking the RG running effect, we **find that UV vacuum (if exists) should be very closely degenerate with the EW vacuum**.
- Taking the potential effects from other scalars, we find that **the Higgs cannot stay at the EW vacuum during inflation**. Also expected tensor-to-scalar ratio is small $r \lesssim 10^{-15}$
- If $U(1)_D$ protected, $m_{DM} \gtrsim 10^{-3}\alpha_D^{1/4}\mathrm{eV}$