Dynamical Solution of the Strong CP Problem

Gerrit Schierholz

Deutsches Elektronen-Synchrotron DESY

With

Yoshifumi Nakamura

RIKEN Center for Computational Science, Kobe

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Objective

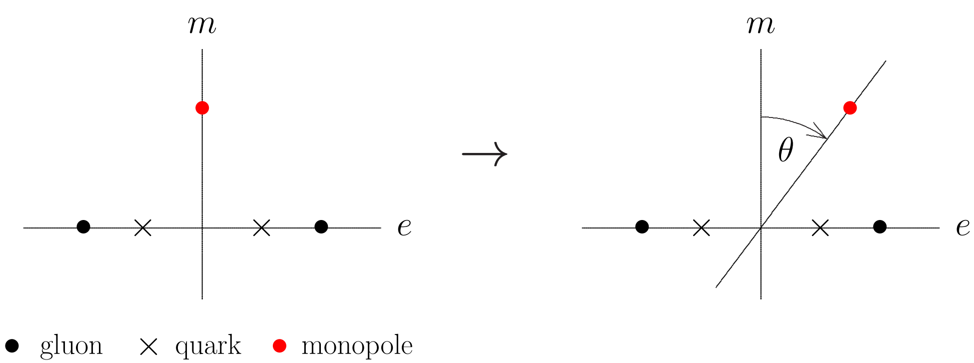
- QCD decribes the strong interactions remarkably well, from the smallest distances probed so far to hadronic scales where quarks and gluons confine to hadrons. Yet it faces a problem. The theory allows for a CP-violating term S_θ in the action. In Euclidean space-time it reads

$$S = S_{\rm QCD} + S_\theta \;:\quad S_\theta = i\,\theta\,Q\,, \quad Q = \frac{1}{32\pi^2}\int d^4x\; F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \;\in\; \mathbb{Z}\,,$$

where Q is the toplogical charge, and θ is an arbitrary phase with values $-\pi < \theta \leq \pi$. A nonvanishing value of θ would result in an electric dipole moment (EDM) d_n of the neutron. The current experimental upper limit is $|d_n| < 1.8 \times 10^{-13} e\,$fm, which suggests that θ is anomalously small. This feature is referred to as the strong CP problem, which is considered as one of the major unsolved problems in the elementary particles field

- The prevailing paradigm is that QCD is in a single confinement phase for $|\theta| < \pi$. The Peccei-Quinn solution of the strong CP problem, for example, is realized by the shift symmetry $e^{i\,\delta\,Q_5}:\; \theta \to \theta + \delta$, trading the theta term S_θ for the hitherto undetected axion

- However, it is known from the case of the massive Schwinger model that a θ term may change the phase of the system. Callan, Dashen and Gross have claimed that a similar phenomenon will occur in QCD. The statement is that the color fields produced by quarks and gluons will be screened by instantons for $|\theta| > 0$. 't Hooft has argued that the relevant degrees of freedom responsible for confinement are color-magnetic monopoles. Confinement occurs when the monopoles condense in the vacuum, by analogy to superconductivity. Due to the joint presence of gluons and monopoles a rich phase structure is expected to emerge as a function of θ



For $|\theta| > 0$ quarks and gluons will be screened by forming bound states with the monopoles

Kronfeld, G.S. and Wiese ; Witten

- In this talk I will investigate the long-distance properties of the theory in the presence of the θ term, S_θ, and show that CP is naturally conserved in the confining phase

Gradient Flow

QCD exhibits a striking change in behavior over different length scales. To reveal the macroscopic properties of the theory, we are faced with a multi-scale problem, involving the passage from the short-distance perturbative regime to the long-distance confining regime. Such multi-scale behavior can only be adddressed by renormalization group (RG) techniques bridging the different regimes

A promising framework is provided by the gradient flow (GF), which evolves the gauge field along the gradient of the action. The flow of SU(3) gauge fields is defined by the diffusion equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

where D_μ is the covariant derivative and $B_\mu(t=0,x) = A_\mu(x)$ is the original gauge field of QCD. It thus defines a sequence of gauge fields parameterized by t. The scale is set by $\mu = 1/\sqrt{8t}$

The gradient flow

— provides a powerful tool for scale setting, with no need for costly ensemble matching, and defines a universal scale μ

— is a particular, infinitesimal realization of the coarse-graining step of momentum space RG transformations and, as such, keeps the long-distance physics unchanged

Lüscher , Suzuki et al.

The force laws underlying the physical system change with respect to the scale μ. Varying μ, the couplings and Green functions satisfy standard RG equations (although depending on the scheme S)

$\alpha_S(\mu)$

The expectation value $\langle E(t) \rangle$ of the energy density

$$E(t,x) = \frac{1}{4} G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$$

has the perturbative expansion

$$\langle E(t) \rangle = \frac{3}{4\pi t^2}\, \alpha_{\overline{MS}}(\mu) \left[1 + k_1\, \alpha_{\overline{MS}}(\mu) + k_2\, \alpha_{\overline{MS}}(\mu)^2 + \cdots \right] \qquad t = 1/8\mu^2$$

$$\equiv \frac{3}{4\pi t^2}\, \alpha_{GF}(\mu)$$

which defines a renormalized coupling in the GF scheme, $\alpha_{GF}(\mu)$, with $\Lambda_{GF} = 1.873\, \Lambda_{\overline{MS}} \ \ (N_f = 0)$

Lüscher

For a start we may restrict our investigations to the Yang-Mills theory. If the strong CP problem is resolved in the Yang-Mills theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \Big(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\,U_{\mu\nu}(x)\Big)$$

	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

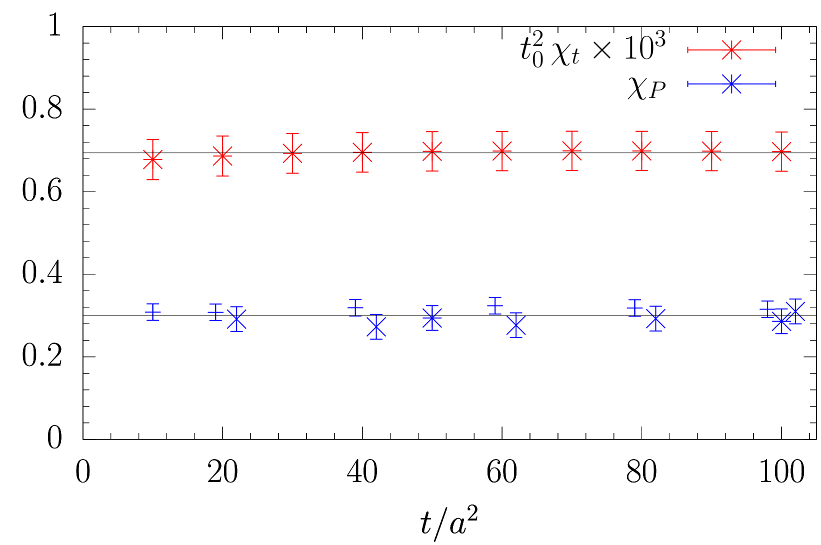
Physical quantities are independent of the RG scale. Two examples: [more to come]

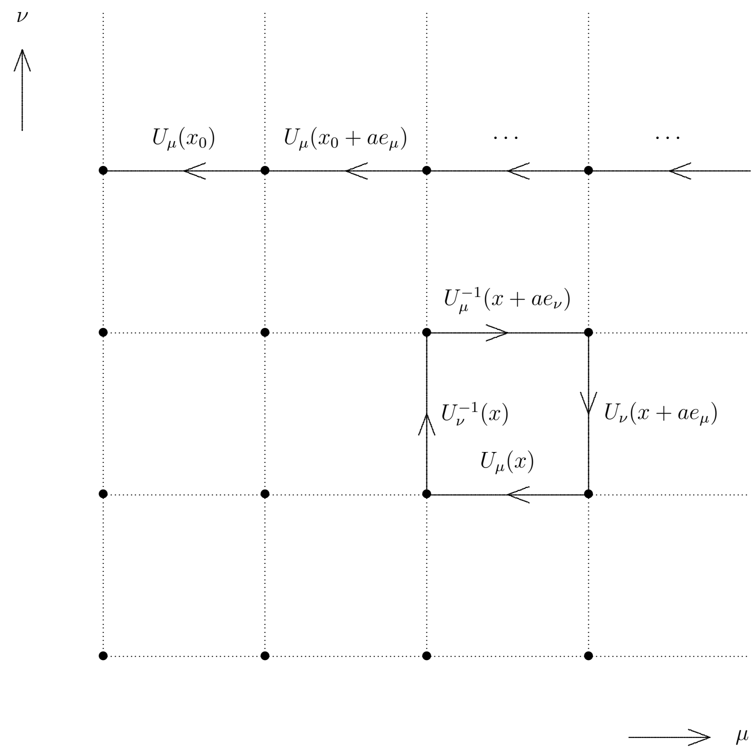
- Topological susceptibility

$$\chi_t = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

- Renormalized Polyakov susceptibility

$$\chi_P = \frac{\langle |P|^2\rangle - \langle |P|\rangle^2}{\langle |P|\rangle^2}\,, \quad P = \frac{1}{V_3}\sum_{\mathbf{x}} P(\mathbf{x})$$





$$U_\mu(x) = \exp\{-iaA_\mu(x + ae_\mu/2)\}$$

$$P(\mathbf{x}) = \frac{1}{3}\,\mathrm{Tr}\prod_{x_0} U_0(x_0, \mathbf{x})$$

$$S = \beta \sum_{x,\mu<\nu}\left(1 - \frac{1}{3}\,\mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}(x)\right)$$

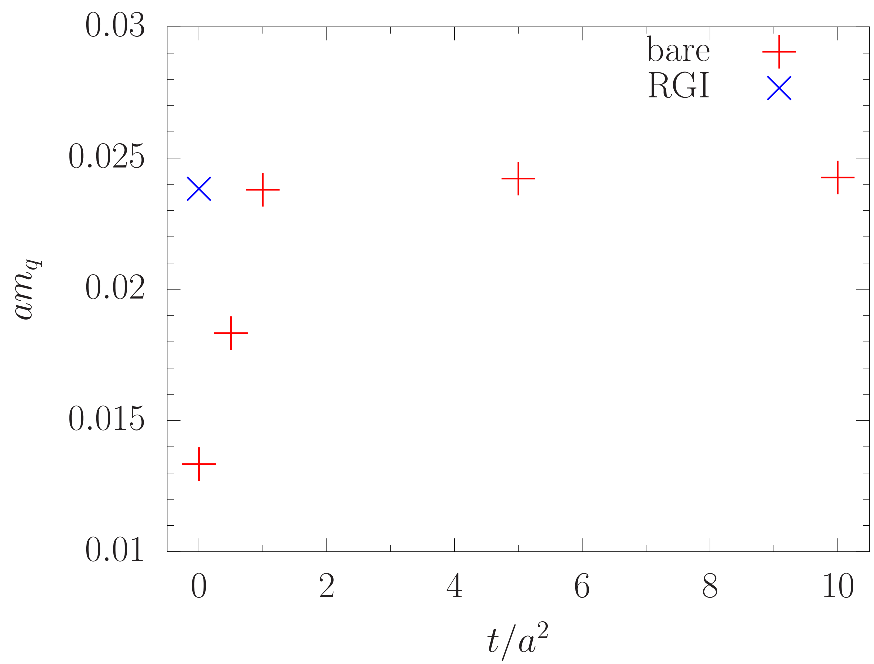
$$= \beta \sum_{x,\mu<\nu} \frac{a^4}{6}\,\mathrm{Tr}\,F^2_{\mu\nu}(x) + O(a^6)$$

$$\simeq \frac{1}{2g^2}\int d^4x \sum_{\mu,\nu}\mathrm{Tr}\,F^2_{\mu\nu}(x)$$

$$\beta = \frac{6}{g^2}$$

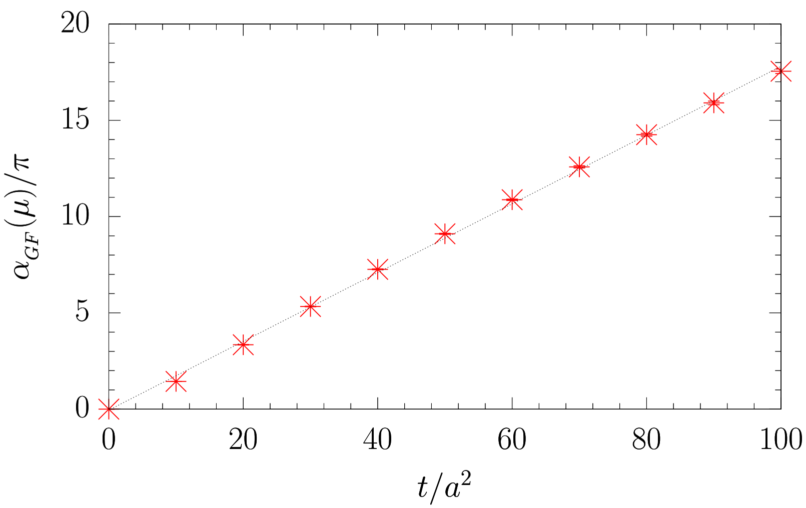
Observables of the flowed field are automatically renormalized

Example: Light quark mass



Confinement

The gradient flow running coupling (at $\mu \to 0$)



$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF}) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -\,2\,\alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0 \alpha_{GF})^{-\frac{b_1}{2\,b_0^2}} \exp\left\{-\frac{1}{8\pi b_0 \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha\, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0 \alpha^2} + \frac{b_1}{2b_0^2 \alpha}\right\}$$

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

To make contact with phenomenology, it is desirable to transform the gradient flow coupling α_{GF} to a common scheme. A preferred scheme in the Yang-Mills theory is the V scheme $V(q) = -4\pi C_F \alpha_V(\mu)/q^2$

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int_0^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int_0^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{q}\; e^{i\,\mathbf{qr}}\,\frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

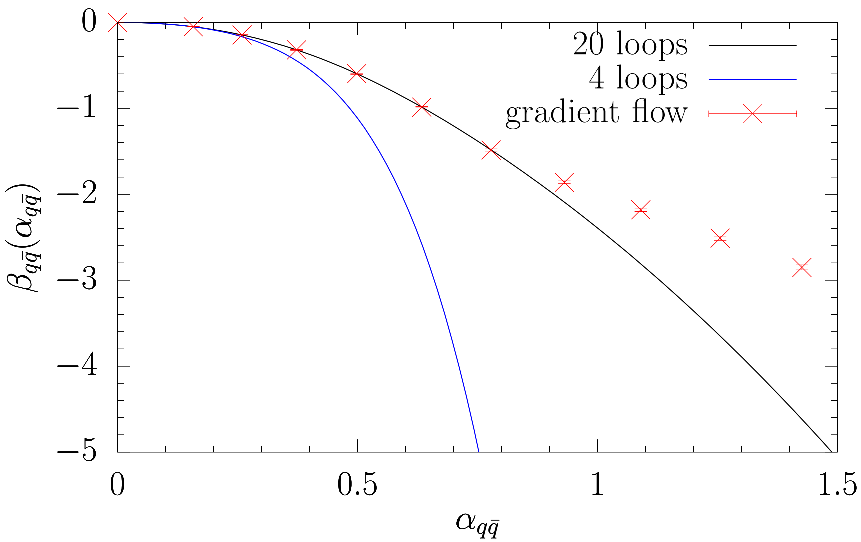
$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

It is interesting to compare the nonperturbative gradient flow beta function with the perturbative beta function known up to twenty loops



20 loops arXiv:1309.4311

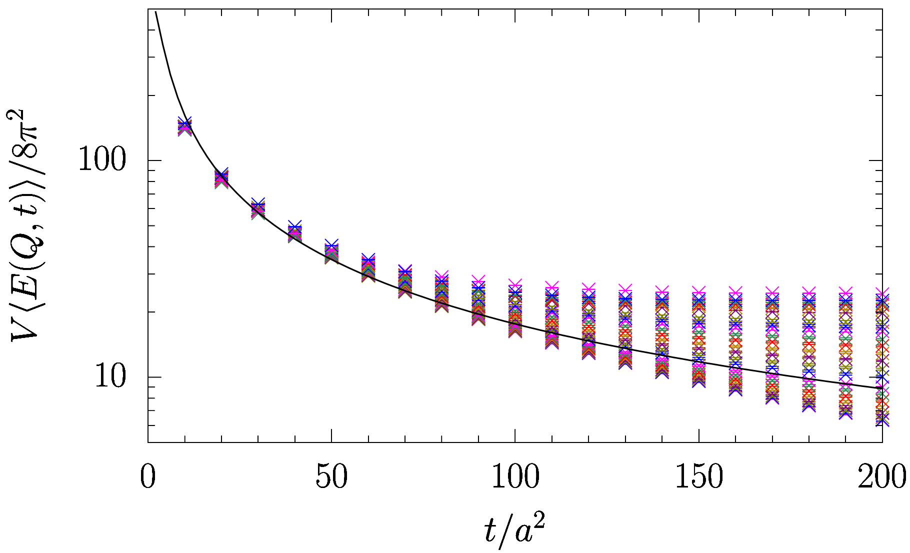
4 loops arXiv:1012.3037

$$\frac{\Lambda_{q\bar{q}}}{\Lambda_V} = 0.655$$

As was to be expected, the perturbative beta function gradually approaches the nonperturbative beta function with increasing order

Phase Structure

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased



$$Z(\theta) = \int \mathcal{D}A_\mu \, e^{-S+i\theta Q}$$
$$= \sum_Q e^{i\theta Q} \int_Q \mathcal{D}A_\mu \, e^{-S}$$
$$= \sum_Q e^{i\theta Q} \, P(Q)$$

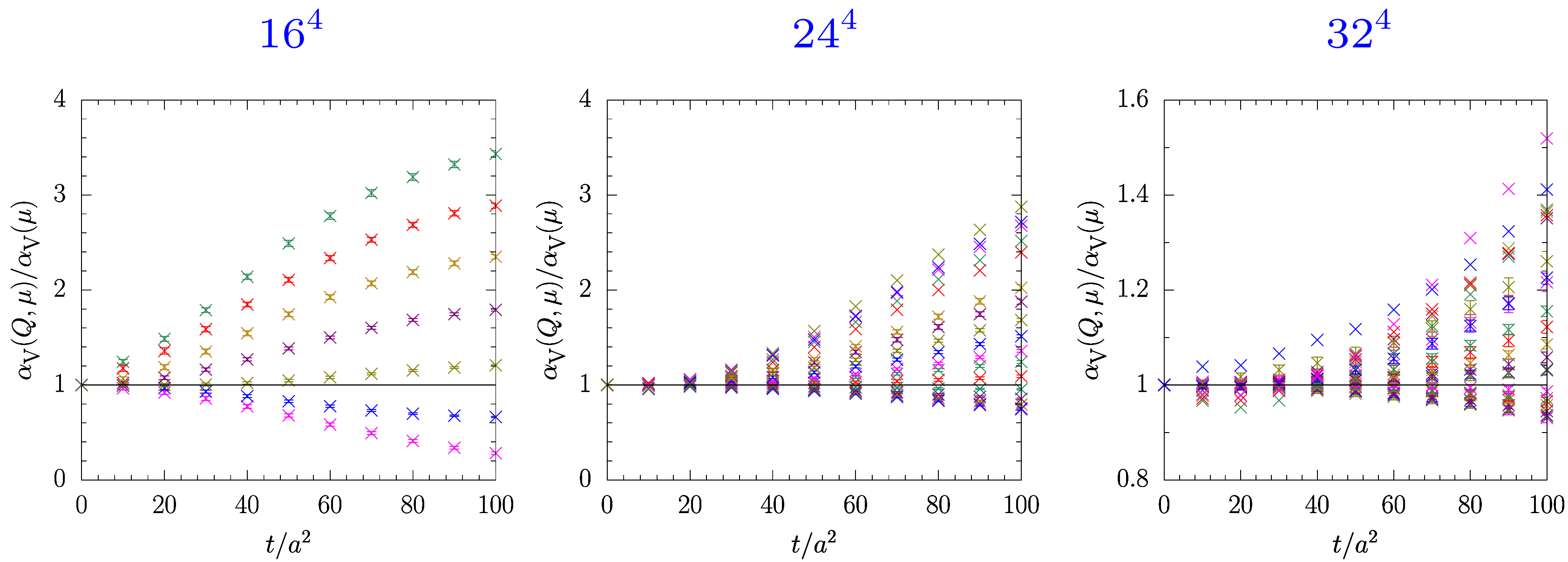
$V\langle E(Q,t)\rangle/8\pi^2 \equiv S_Q \simeq |Q|$, while the ensemble average vanishes like $1/t$

Observables consistently show a clear dependence on Q. This is the reason for a nontrivial θ dependence when Fourier transformed to the θ vacuum

Running coupling α_V

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top



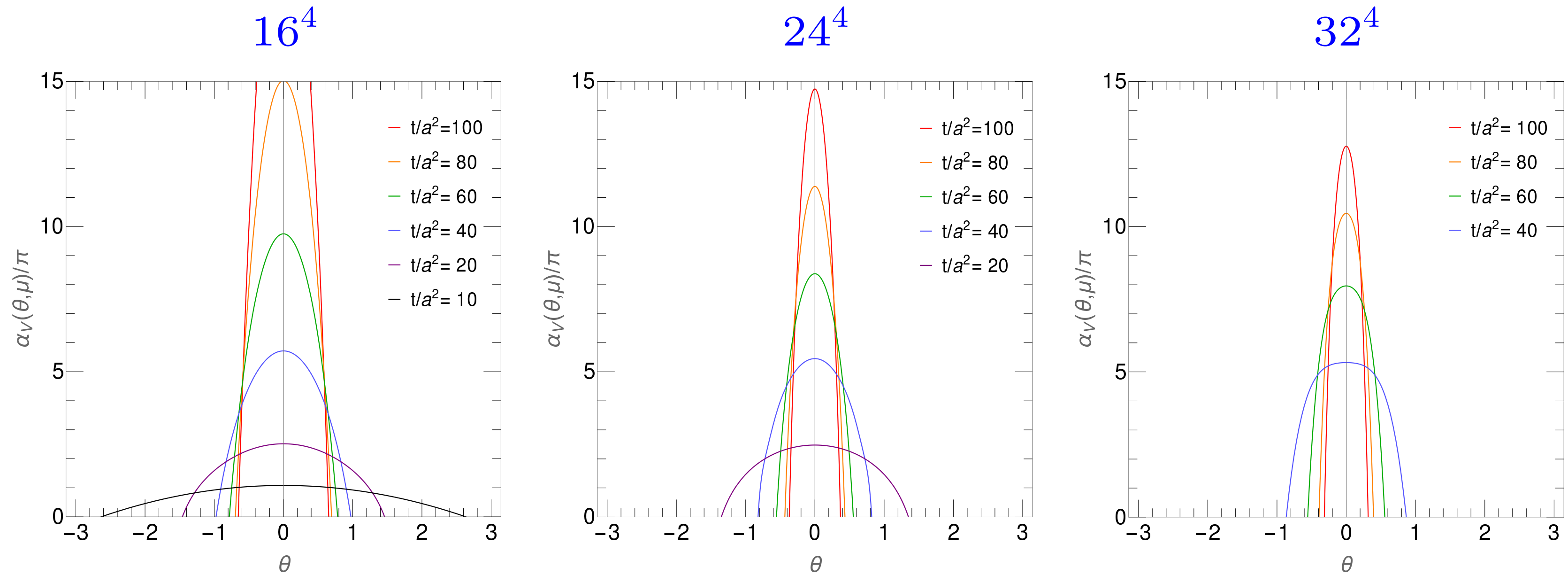
Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

The transformation of $\alpha_V(Q,\mu)$ from Q to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q} P(Q) \alpha_V(Q,\mu)$$

$$Z(\theta) = \sum_Q e^{i\,\theta\,Q} P(Q)$$

- $e^{i\theta Q} P_{\theta=0}(Q) = P_\theta(Q)$ 1502.02295
- Z_θ analytic at $\theta = 0$ Vafa-Witten
- Limits set by convergence of the Fourier sum

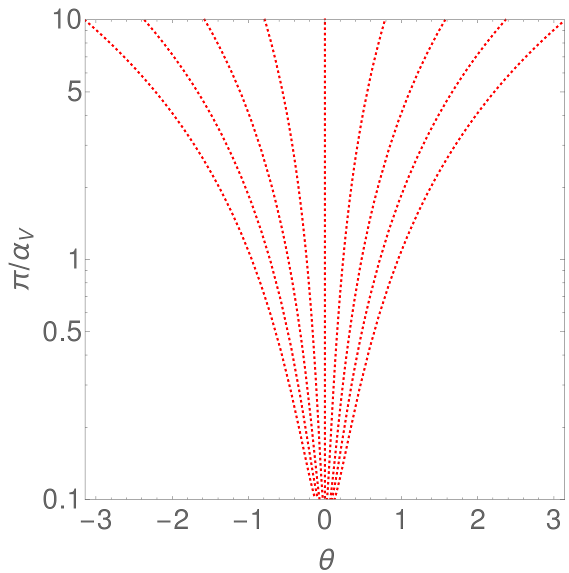


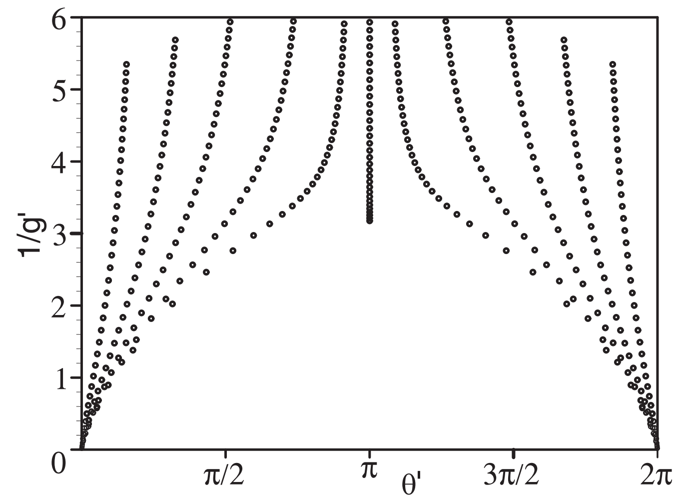
The color charge is totally screened for $|\theta| \gtrsim 0$ in the infrared, while it becomes gradually independent of θ as we approach the perturbative regime

Behavior for $|\theta| > 0$ similar to $\alpha_s(T,\mu)$ at $T > T_c$

It is tempting to derive RG flow equations for the running coupling constant $\alpha_V(\theta, \mu)$, by analogy with the quantum Hall conductivity

Quantum Hall effect (model)





$$\frac{\partial(\pi/\alpha_V)}{\partial \ln t} \simeq \frac{\pi}{\alpha_V} + D\theta^2 \quad \frac{\partial\theta}{\partial \ln t} \simeq -\frac{1}{2}\theta$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{pmatrix}$$

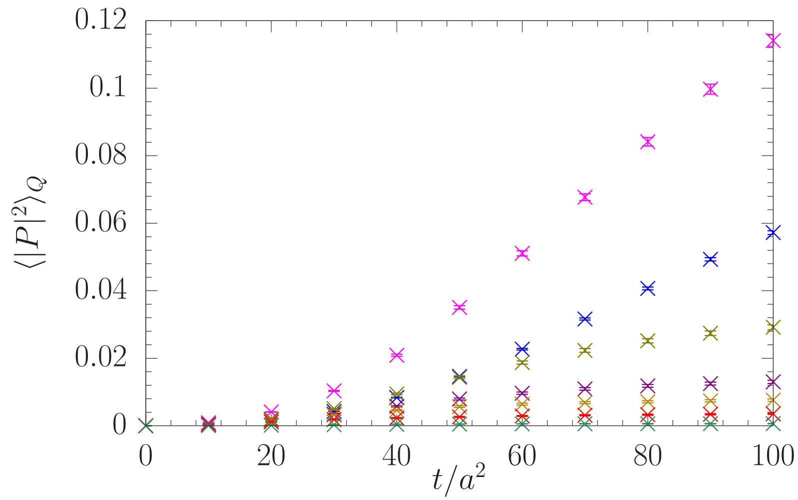
$$\sigma_{xx} \sim 1/g^2 \qquad \sigma_{xy} \sim \theta$$

Knizhnik & Morozov , Levine, Libby & Pruisken , Apenko

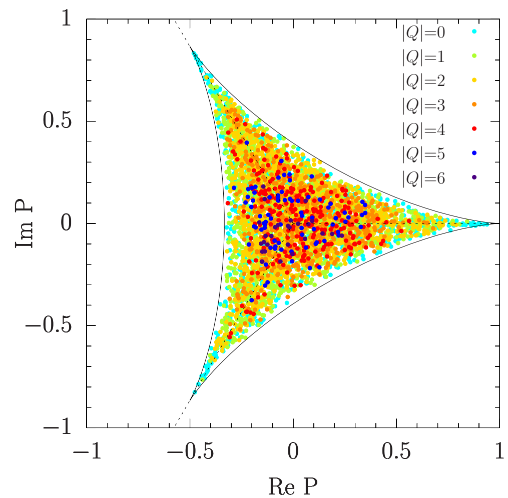
Polyakov loop

The Polyakov loop P describes the propagation of a single static quark travelling around the periodic lattice

16^4



$t/a^2 = 60$



From $Q = 0$ (top) to 6 (bottom)

$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

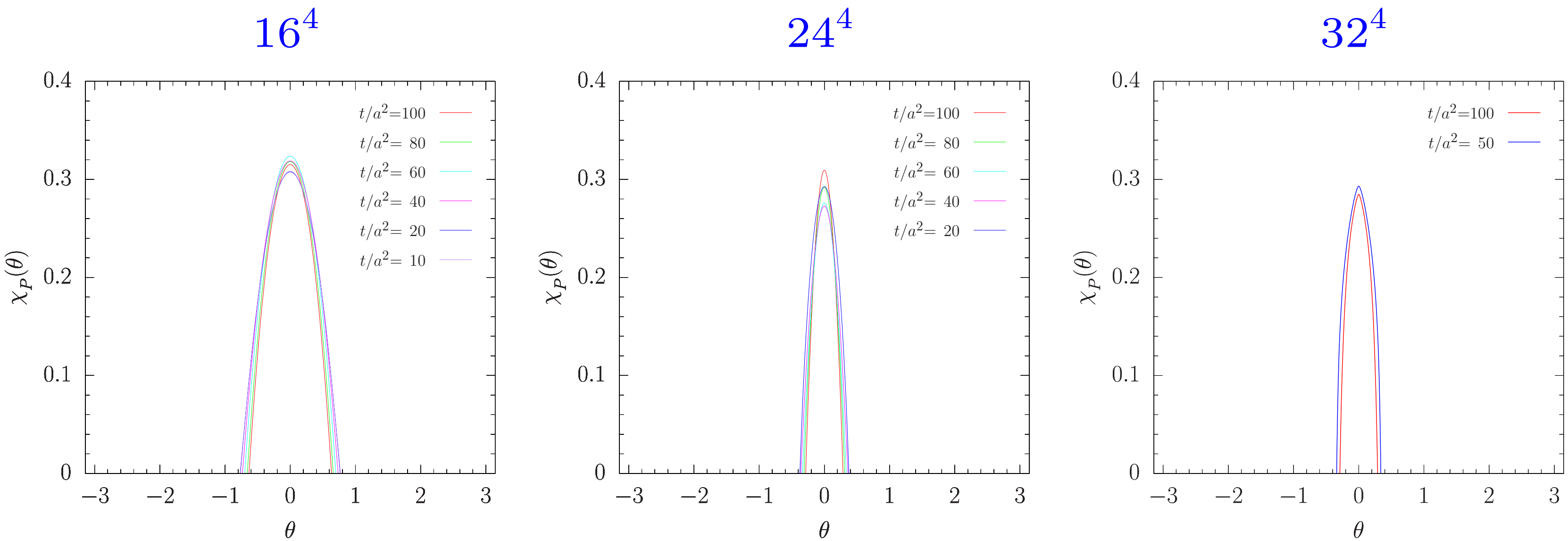
The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The connected part of $\langle |P|^2 \rangle_\theta$ is described by the renormalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The renormalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

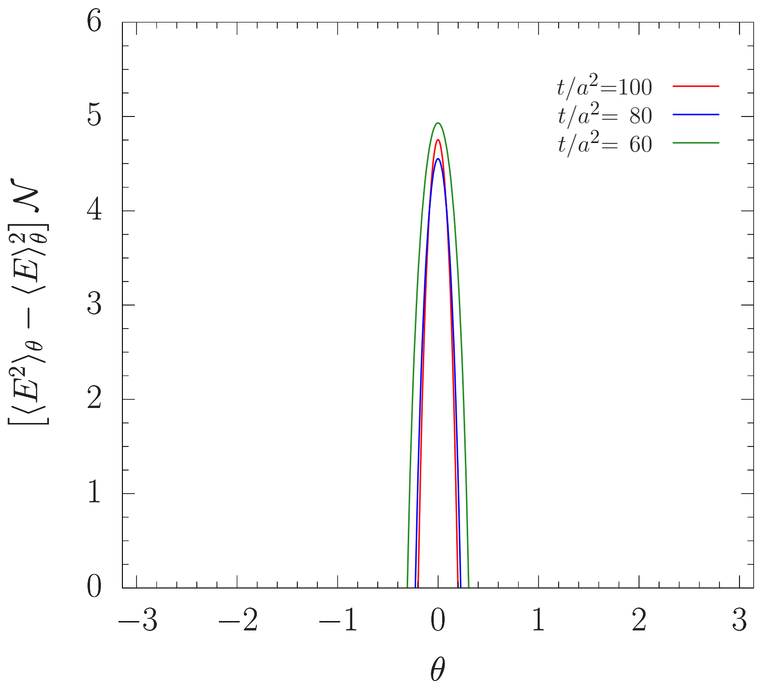
Mass gap

$$\langle E^2 \rangle = \frac{1}{T} \sum_t \langle E(0) E(t) \rangle \qquad [\langle E^2\rangle - \langle E\rangle^2]\,\mathcal{N} = \sum_{n>0,t} \frac{1}{2m_n} |\langle 0|E|n\rangle|^2 e^{-m_n t}$$

$$E(t) = \frac{1}{V_3} \sum_{\vec{x}} E(\vec{x}, t) \qquad \simeq \frac{1}{m_{0^{++}}^2} |\langle 0|E|0^{++}\rangle|^2 \propto \xi^2$$

Correlation length

24^4



$\langle E^2\rangle_\theta - \langle E\rangle_\theta^2$ Independent of flow time t

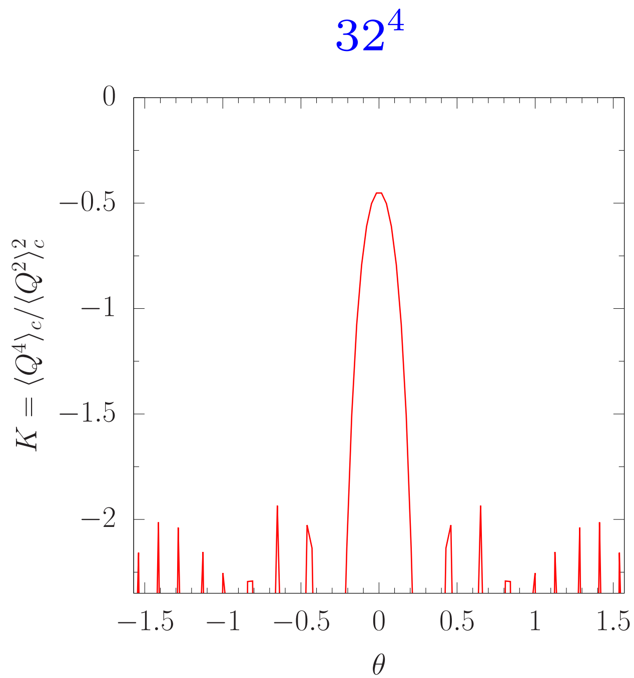
$\xi \simeq 0$ for $|\theta| \gtrsim 0$

No mass gap

Kurtosis

The key to understanding the transition to nonvanishing θ lies in the topological structure of the vacuum. Our global analysis limits us to the investigation of moments of topological charge Q. A quantity of particular interest is the kurtosis K. In the θ vacuum

$$K = \frac{\langle (Q - \langle Q \rangle_\theta)^4 \rangle_\theta}{\langle (Q - \langle Q \rangle_\theta)^2 \rangle_\theta^2} - 3 \equiv -V \frac{\partial^4 F(\theta)/\partial\theta^4}{(\partial^2 F(\theta)/\partial\theta^2)^2}, \qquad F(\theta) = -\frac{1}{V} \log Z(\theta)$$



The lowest value the kurtosis can take is $K = -2$, that is when

$$\langle (Q - \langle Q \rangle_\theta)^4 \rangle_\theta = \langle (Q - \langle Q \rangle_\theta)^2 \rangle_\theta^2$$

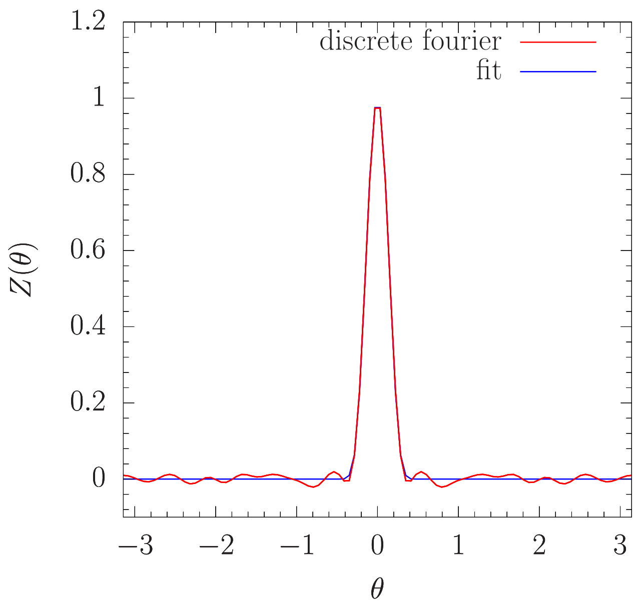
which means that the topological charge Q is screened on its own:

$$\chi_{t,\theta} = \frac{\partial^2 F(\theta)}{\partial\theta^2} = \frac{1}{V}\, \wp(\theta; 0, g_3), \ |\theta| \gtrsim 0$$

Errors

Source of errors

- Convergence of the (discrete) Fourier series $\sum_Q \exp\{i\theta Q\}\, P(Q) \cdots$
- Statistics
- Topological charge generally limited to $|Q| \le |Q|_{\max}$, $|Q|_{\max} \propto \sqrt{V}$



$Z(\theta), \alpha_V(\theta), \chi_P(\theta), \cdots$ are positive functions of θ

After the quantities I showed have dropped to 'zero' at $|\theta| \gtrsim 0$, they start to oscillate around zero with frequency $\nu \approx |Q|_{\max}$ due to the truncated Fourier series

Various techniques to filter unphysical high-frequency modes are discussed in the literature. We fit the tail of the distributions to a smooth function. Alternatively, one can employ a low-pass filter, which practically gives the same result

EDM & Axion

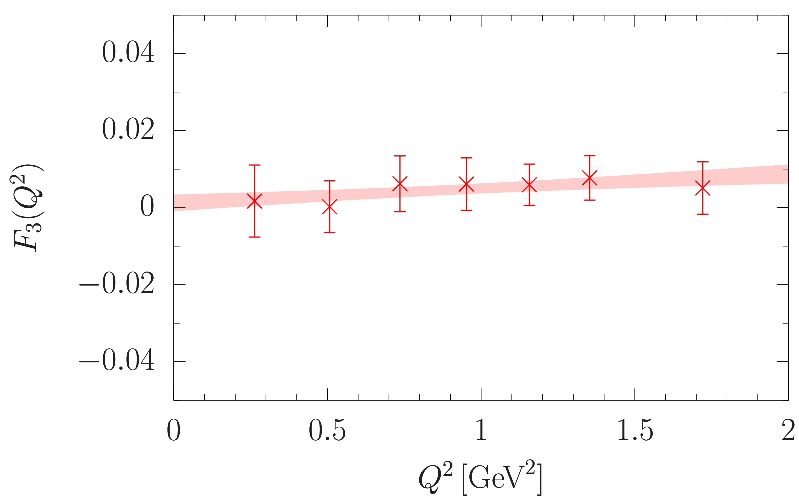
EDM

We expect the electric dipole moment of the neutron to be largest for heavy quarks, as it will vanish trivially in the chiral limit, $d_n \propto m_u m_d/(m_u + m_d)$

At the SU(3) flavor symmetric point, $m_\pi = m_K = 410$ MeV

arXiv:1102.5200

$32^3 \times 64$



$|\theta| \approx 0.4$

This leads to

$$d_n = \frac{e\, F_3(0)}{2m_N} = 0.00028(30) \text{ [e fm } \theta]$$

which is compatible with zero, as expected

Similar results have been reported for using reweighting

arXiv:1701.07792 , 2011.01084 , 2101.07230

Axion

In the Peccei–Quinn theory the CP violating action $S_\theta = i\theta Q$ is augmented by the axion interaction

$$S_\theta \to S_\theta + S_{\rm Axion} = \int d^4x \left[\frac{1}{2}\big(\partial_\mu \phi_a(x)\big)^2 + i\left(\theta - \frac{\phi_a(x)}{f_a}\right)\, q(x)\right], \quad \int d^4x\, q(x) = Q$$

with

$$U_{\rm PQ}(1): \quad e^{i\delta Q_5}\,|\theta\rangle \qquad \longrightarrow \qquad |\theta + \delta\rangle$$

It is then expected that QCD induces an effective potential $U_{\rm eff}(\theta - \phi_a/f_a)$, having a stationary point at $\theta - \phi_a/f_a = 0$, which prompts the field redefinition $\phi_a \to \phi_a + f_a\,\theta$:

$$\theta \qquad \longrightarrow \qquad \frac{\phi_a(x)}{f_a}$$

CP violating   CP conserving

thus effectively eliminating CP violation in the strong interaction

However, QCD vacuum unstable under $U_{\rm PQ}(1)$

Conclusions

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. A key point is that the path integral splits into disconnected topological sectors for $t \gtrsim 0$, which is expected to occur at ever smaller flow times with decreasing lattice spacing. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The novel result is that color charges are screened for $|\theta| > 0$ by nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level

★ Screening is a gradual process, most likely a crossover, which is completed once the vacuum has attained a sufficient level of color-electric charge density, which seems to be the case for $|\theta| \gtrsim 0.5$. This result does not come as a surprise. One simply did not have the tools to address the problem

★ The electric dipole moment of the neutron was found to be zero within the errorbars, as expected. In absence of a nonvanishing dipole moment no upper limit of θ can be drawn from the experimental bound

★ The nontrivial phase structure of QCD has far-reaching consequences for anomalous chiral transformations. In particular, the confining QCD vacuum will be unstable under the Peccei-Quinn chiral $U_{\mathrm{PQ}}(1) = e^{i\delta Q_5}$ transformation, realizing the shift symmetry $\theta \to \theta + \delta$, which thwarts the axion conjecture